The Journal of

THE ROYAL AERONAUTICAL SOCIETY

SEPTEMBER 1947 NO 441 VOL. 51

He knows where he is with Kelvin Instruments . . . he knows that a Kelvin pointer, moving across a Kelvin dial, will give him accurate information . . . clearly . . . quickly . . . whenever he needs it. Kelvin reliability is built on years of accumulated experience and organised research. It is backed by a tradition of accuracy inspired by the genius of Lord Kelvin, master of measurement. That is why Kelvin aircraft instruments are being specified for high speed aircraft now being built in this country.

KELVIN AIRCRAFT INSTRUMENTS

PROVEN IN RELIABILITY • AHEAD IN DESIGN

KELVIN BOTTOMLEY AND BAIRD LIMITED • BASINGSTOKE

THE ROYAL AERONAUTICAL SOCIETY
4 HAMILTON PLACE, PICCADILLY, LONDON, W.1

SEPTEMBER NOTICES 1947

JOURNAL PREMIUM AWARDS

The Council have set aside an annual sum of £250 for the award of premiums for papers published in the *Journal* and the Council hope that members (or non-members) will contribute papers on their own special subjects.

GARDEN PARTY

Members are reminded that the Garden Party on 14th September 1947, at the Handley Page Aerodrome, Radlett, will **begin at 2.30 p.m. and will end at 7 p.m.**

Car parks will not be open, nor will members be admitted to the aerodrome, before 1.45 p.m. Members and their guests are requested to wear their badges to help the staff organisation.

CONTENTS OF THE SEPTEMBER JOURNAL

The Behaviour of Structures Under Repeated Loads, by A. G. Pugsley, O.B.E., D.Sc., F.R.Ae.S.

Gusts, by Walter Tye, B.Sc., F.R.Ae.S.

High Speed Aircraft Flying Limitations and Handling Problems, by F. W. Morgan, B.Sc., A.R.Ae.S.

The General Theory of Cylindrical and Conical Tubes under Torsion and Bending Loads, Part V, by J. Hadji-Argyris, D.E., A.F.R.Ae.S., and P. C. Dunne, B.Sc., A.F.R.Ae.S.

S.B.A.C. SCHOLARSHIPS

The following have been awarded scholarships by the Selection Committee of the Society of British Aircraft Constructors and the Royal Aeronautical Society:—

D. Fletcher—S.B.A.C. Educational Grant, apprenticeship at Saunders-Roe, Isle of Wight;

D. I. Powell—S.B.A.C. Educational Grant, apprenticeship at the Fairey Aviation Co. Ltd., Hayes;

R. H. Hayward—John de Havilland Scholarship, apprenticeship at The de Havilland Aircraft Co. Ltd., Hatfield.

HONOURS

Dr. H. Roxbee Cox, President-Elect, has been presented with the American Medal of Honour with Silver Palm at a ceremony at the American Embassy. The presentation was made by Major-General Clayton L. Bissell. The citation states that Dr. Roxbee Cox rendered exceptionally meritorious service to the Government of the United States, from September 1941 to September 1945, in connection with the development of turbo-jet and turbine aircraft engines.

SYMPOSIUM ON INTERNAL STRESSES IN METALS

A Symposium on internal stresses in metals and alloys has been arranged by the Institute of Metals in association with the Faraday Society, the Institute of Physics, the Institution

of Mechanical Engineers, the Iron and Steel Institute, the Physical Society and the Royal Aeronautical Society.

The Symposium will be held on **Wednesday and Thursday, 15th and 16th October 1947,** at the Institution of Mechanical Engineers. Thirty-six papers have been received and a limited number of advance copies of the papers will be available at a charge of 5s. (post free). Because of the shortage of paper it is particularly requested that those who are not able to take part in the meeting will await publication of the final report.

The Symposium will be divided into three main sections. The first, under the chairmanship of Dr. C. Sykes, F.R.S., on The Measurement of Internal Stresses; the second on The Origin, Control and Removal of Internal Stresses, under the chairmanship of Professor L. Aitchison, D.Met., B.Sc., F.R.Ae.S.; and the third on Effects Associated with Internal Stresses, under the chairmanship of Dr. Maurice Cook and Dr. H. J. Gough, C.B., M.B.E., F.R.S.

The first part begins at 10 a.m. on 15th October 1947, and ends at 12.45; the second at 2.30 and ends at 5.0 p.m., and the third at 10 a.m. on 16th October and ends at 5.0 p.m.

Further particulars can be obtained from The Secretary, Institute of Metals, 4 Grosvenor Gardens, S.W.1.

BIRTHDAY HONOURS

Lt. E. Grocock (A) R.N., Associate, was awarded the M.B.E. (Military Division) in the Birthday Honours.

LECTURE PROGRAMME—AUTUMN SESSION 1947

The Lectures will be held at 6 p.m. in the Lecture Hall of the Institution of Civil Engineers, Great George Street, S.W.1 (by permission of the Council of the Institution). Tea will be served at 5.30 p.m.

Visitors are welcome, but should obtain tickets through a member of the Society.

Thursday, 9th October 1947—Pressurisation of Aircraft, by W. M. Widgery, F.R.Ae.S.

Thursday, 23rd October 1947—The Problem of High Temperature Alloys for Gas Turbines, by Sir William T. Griffiths, D.Sc., F.R.I.C., F.Inst.P., F.I.M.

Thursday, 30th October 1947—The Third British Commonwealth and Empire Lecture, by James Bain of Trans-Canada Airlines.

Thursday, 6th November 1947—Some recent Developments in the Landing Gear Field, by Captain R. Lucien, A.F.R.Ae.S.

Thursday, 20th November 1947—The Development of the A.S. Type Gas Turbine, by W. H. Lindsey, M.A., A.F.R.Ae.S.

Thursday, 4th December 1947—Problems Facing Civil Airline Operations, by N. E. Rowe, C.B.E., B.Sc., D.I.C., F.R.Ae.S.

Thursday, 18th December 1947—The Work of the High-Speed Tunnel, by Professor A. Thom, M.A., D.Sc., and W. G. A. Perring, F.R.Ae.S.

GLASGOW BRANCH

Mr. A. Wauchope Fraser, A.R.Ae.S., has been appointed Secretary of the Glasgow Branch in succession to Mr. D. R. Briant. Mr. Fraser's address is: 39 West Crescent, Muirhead, Troon, Ayrshire.

PORTSMOUTH BRANCH

26th September 1947—Model Section Display, at the Cosham Civic Centre at 7 p.m.

GRADUATES' AND STUDENTS' SECTION

Meetings

Tuesday, 23rd September 1947—Discussion on the Design of Wings, introduced by Mr. F. Tyson, Assistant Chief Stressman, Handley Page Ltd.

Tuesday, 21st October 1947—Lecture on Rocket Propulsion and Interplanetary Flight, by Mr. A. V. Cleaver, A.R.Ae.S., Special Projects Engineer, The de Havilland Engine Company Ltd.

Meetings will be held in the Library, 4 Hamilton Place, W.1, at 7.30 p.m.

Visit

17th September 1947 (Wednesday)—A visit has been arranged to the works of the de Havilland Aircraft Co. Ltd. at 10.30 a.m. Application to attend should be made to the Hon. Secretary, J. G. Roxburgh, 62 Fitzjohns Avenue, London, N.W.3.

Dance

The Annual Dance, open to members and their friends, will be held on Friday, 17th October 1947 at the Royal Hotel, Woburn Place, W.C.1 (nearest Tube Station, Russel Square—Piccadilly Line). Tickets, price 5/- each, may be obtained from the Honorary Secretary, J. G. Roxburgh, 62 Fitzjohns Avenue, London, N.W.3, or from the Section's representatives at firms and colleges.

SPECIAL GENERAL MEETINGS

The new Statutes and Rules of the Society were passed at the Special General Meeting held on 23rd January 1947, and the adjourned meeting of 25th April 1947.

The following are the rules then passed for joining the Society as a Fellow, Associate Fellow, Associate, Graduate, Student or Companion:—

4. **Fellows** shall comprise every person who was on the register on January 23rd 1947 as a Fellow of the Society; and every person thereafter awarded the honour of Fellowship.

Awards of Fellowship will be made annually by the Council and will be announced at the Annual General Meeting of the Society. Selections will be initiated by the Council normally only from among those who have been for at least five years members of the Society. It is the duty of the Council to see that the honour is awarded only to persons who have attained a considerable degree of technical eminence in the profession of aeronautics.

5. **Associate Fellows** shall comprise every person who was on the register on January 23rd 1947 as an Associate Fellow of the Society and every person thereafter elected or transferred into the class of Associate Fellow.

Every candidate for election into the class of Associate Fellow shall possess the following qualifications:—

(*a*) He shall be twenty-five years of age or over.

(*b*) He shall be actually engaged at the time of his application for election in the profession of aeronautics.

(*c*) He shall have passed the Associate Fellowship Examination of the Society. The Council may exempt a candidate therefrom to the extent to which he has passed other examinations recognised by them.

(*d*) (Alternative to (*c*).) He shall have had at least fifteen years' practical experience in the profession of aeronautics, shall be forty years of age or over, and have had a record of distinguished service. His nomination must be approved by four Fellows of the Society.

(*e*) He shall have had in addition to his period of training five years' experience in the profession of aeronautics.

6. **Associates** shall comprise every person who was on the Register on January 23rd 1947 as an Associate of the Society and every person thereafter elected or transferred into the class of Associate.

Every candidate for election into the class of Associate shall possess the following qualifications:—

(*a*) He shall be twenty-five years of age or over.

(*b*) He shall be actually engaged at the time of his application for election in the profession of aeronautics.

(*c*) He shall produce satisfactory evidence of his technical knowledge in the profession of aeronautics.

(*d*) He shall have had in addition to his period of training five years' experience in the profession of aeronautics.

7. **Graduates** shall comprise every person who was on the Register on January 23rd 1947 as a Graduate of the Society and every person thereafter elected or transferred into the class of Graduate provided that they shall cease to remain on the Register as a Graduate of the Society when they cease to qualify under section (*a*) as follows.

Every candidate for election into the class of Graduate shall possess the following qualifications:—

(*a*) He shall be twenty-one years of age or over, but shall be not more than twenty-eight years of age (except in such cases as the Council may determine).

(*b*) He shall satisfy the Council that he is engaged in or receiving training in the profession of aeronautics.

(*c*) He shall have passed the Associate Fellowship Examination of the Society. The Council may exempt a candidate therefrom to the extent to which he has passed other examinations recognised by them.

8. **Students** shall comprise every person who was on the Register on January 23rd 1947 as a Student of the Society and every person thereafter elected into the class of Student provided that they shall cease to remain on the Register as a Student of the Society when they cease to qualify under section (*a*) below.

Every candidate for election into the class of Student shall possess the following qualifications:—

(*a*) He shall be eighteen years of age or over, but shall be not more than twenty-five years of age (except in such cases as the Council may determine).

(*b*) He shall satisfy the Council that he has the object or intention of becoming engaged in the profession of aeronautics.

(*c*) He shall have passed the Common Preliminary or an exempting examination.

9. **Companions** shall comprise every person who was on the Register on January 23rd 1947 as a Companion of the Society and every person thereafter elected or transferred into the class of Companion.

Companions shall be those persons who being ineligible for the technical grades, have contributed or are likely to contribute, to the development of aeronautical science and engineering and to the furtherance of the objects of the Society.

On 7th August 1947, a Special General Meeting was called to enable the President and Vice-Presidents, elected in May and June 1947 under the rules then operative, to remain in office until the May meeting of the Council in 1949. The resolution to this effect was carried unanimously.

A second resolution to alter the present rates of the Annual Subscriptions of members was not carried by the necessary two-thirds majority.

ASSOCIATE FELLOWSHIP EXAMINATION DECEMBER 1947

Candidates for the Associate Fellowship Examination are reminded that entries for the December examination should be received by the Secretary by 30th September 1947.

THE NEW SYLLABUS FOR ASSOCIATE FELLOWSHIP EXAMINATIONS

The new syllabus for the Associate Fellowship Examinations will come into force on 1st January 1948. The syllabus is now in the press and is expected to be available by 1st October 1947.

CHANGES OF ADDRESS

To assist in keeping the records of members correct and up to date the Secretary will be glad if all members will notify him as soon as possible of changes of address.

When notifying changes please give the following particulars:—

Name (in block letters).
Grade of membership.
New address (in block letters).
Old address.

Changes of address must be received *before the 15th of the month* in order to be effective for the Journal for the following month.

ELECTION OF MEMBERS

The following new members have been elected:—

Fellow

Archibald Buchanan Barbour (from Associate Fellow).

Associate Fellows

Edgar Parsons Bridgland (from Graduate), Alexander Pearse Johnstone, Harold Murphy (ex-Student), Salem Gopalaswamy Rangaswamy, William Robert Oliver Sellick.

Associates

Ulryk Breier, Benjamin Revett Cant, Charles John Edwin Carter, William Cunniffe, Jack Hardy, Munir Helmi, Clifford George Hollowell.

Graduates

Robert Frank Brown (from Student), Ian Wedderburn Clunie Robertson, Frederick Danesbury Raper, Patrick James Sibley (from Student).

Students

John Russell Baxter, Dennis Leslie Campion, Leslie Vincent Chapman, Francis Farrell Heaton, Henry Trevor Howard, John W. Jones, James Antony Rowland, Robert James Skillen.

ACKNOWLEDGMENTS

The Council acknowledge with grateful thanks the following gifts: A film of a Dornier Do.X flying boat being inspected by H.R.H. The Prince of Wales in 1930, from Mr. G. F. Webb, A.F.R.Ae.S., of the Medway Branch; and back numbers of the Journal from W. A. J. Wall, A.F.R.Ae.S., and E. E. H. Evans, A.F.R.Ae.S.

The Council also acknowledge the gift of photographs of Wilbur Wright and his aeroplane at Pau in 1909, from Dame Katherine Furse.

ADDITIONS TO THE LIBRARY

Pamphlets in italics with location reference following in brackets. Books marked * or ** may not be taken out on loan.

B.b.27—Pioneering the Helicopter. Charles Lester Morris. McGraw Hill. 1945.

B.e.74—Gliding and Power Flying. "Stringbag." Oxford University Press. 1947.

BB.b.129—Experimental Study of Structures. A. J. S. Pippard. Arnold. 1947.

D.a.95—*Wings over the world. Annual Report* 1946. *Pan American World Airlines.* 1947.

D.b.271—International Air Transport. Lord Sempill (Ed.). Todd.

D.c.116—Airports: Design, Construction and Management. H. K. Glidden, H. F. Law and J. E. Cowles. McGraw Hill. 1946.

E.d.32—Aircraft Layout and Detail Design. N. H. Anderson. McGraw Hill. 1946.

EE.a.126—Handbook for the Siddeley "Puma" 240 B.H.P. aircraft engine. October 1917. Palethorpe.

EE.a.127—*Report on the* 260 *h.p. Mercedes aero-engine. July* 1917. *Technical Information Section, Air Board.* (Y.7.)

G.a.78—Metallurgy. R. A. Beamont. Pitman. 1946.

K.c.29—Rockets. Robert H. Goddard. American Rocket Society. 1946.

L.d.116—*Notices to Airmen. Nos.* 190, 191,,192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 213, 214, 215, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228.

L.d.120—Manual of Celestial Navigation. A. A. Ageton. van Nostrand, New York. 1947.

L.j.97—Manual of Photogrammetry. P. G. McCurdy and others (Ed.). Pitman. 1944.

L.k.64—Biology of Flight. F. L. Fitzpatrick and K. A. Styles. Allen & Unwin. 1944.

M.b.55—Reference Data for Radio Engineers. W. L. McPherson. Standard Telephones and Cables Ltd.

*Q.a.175—Supplementary Bibliography on Powder Metallurgy. August 1946-July 1947. Institute of Metals.

*Q.b.75a—Supplement to catalogue of Lewis' Medical, Scientific and Technical Lending Library.

R.d.41—*A historical summary of the Royal Aircraft Factory and its antecedents.* 1878-1918. *S. Child and C. F. Caunter. R.A.E.* 1947.

S.b.153—The R.C.A.F. Overseas. Fifth Year. Colin Gibson (Ed.). Oxford University Press, Toronto. 1945.

T.b.103—Transocean Flight Attempts. Arthur Flury (comp.). Published by the author, Bern. 1947.

UE.5/35—*C.S.I.R. Division of Aeronautics. Report S.M.*78. *An introduction to relaxation methods, approximate methods of numerical computation. F. S. Shaw.*

X.c.158—Aeronautical Dictionary. T. A. Dickenson. Pitman. 1946.
X.c.67—Soaring Society of America Inc. Roster. January 1947.

A.R.C. Reports and Memoranda

2111—*The design of suction aerofoils with a very large C1-range. M. B. Glauert.*

2078—*The disturbed longitudinal motion of an uncontrolled aircraft and of an aircraft with automatic control. F/Lt. S. Neumark.*

2094—*The theory of thin plates under transverse pressure. A. G. Pugsley and D. M. A. Leggett.*

2138—*High-speed tunnel measurement of fin-tailplane interference between EC 1040 and EC 1240 sections. W. F. Hilton, S. J. Moore and R. F. Sargent.*

2107—*The effect of the boundary layer and wake on the flow past a symmetrical aerofoil at zero incidence. Part I: The velocity distribution at the edge of, and outside the boundary layer and wake. J. H. Preston.*

2088—*Determination of the optimum twist of an airscrew blade by the "Calculus of Variations." C. N. H. Lock, R. C. Pankhurst and R. G. Fowler.*

2137—*Note on wind-tunnel contractions. F. Cheers.*

2084—*Interference velocity for a close pair of contra-rotating airscrews. C. N. H. Lock.*

2142—*Theoretical air-load and derivative coefficients for rectangular wings. W. Prichard Jones.*

2145—*Theoretical determination of the pressure distribution on a finite wing in steady motion. W. Prichard Jones.*

N.A.C.A. Technical Notes

1113—*Collection and analysis of hinge-moment data on control-surface tabs. Paul E. Purser and Charles B. Cook.*

1122—*Sideslip angles and vertical-tail loads in rolling pull-out manœuvers. Maurice D. White, Harvard Lomax and H. L. Turner.*

1161—*Effect of catalysts and pH on strength of resin-bonded plywood. G. M. Kline, F. W. Reinhardt, R. C. Rinker and N. J. de Hollis.*

1166—*Variation of hydrodynamic impact loads with flight-path angle for a prismatic float at 0 degrees and −3 degrees trim and with a 22½ degrees angle of dead rise. Sidney A. Batterson.*

1168—*Some recent contributions to the study of transition and turbulent boundary layer. Hugh L. Dryden.*

1171—*On subsonic compressible flows by a method of correspondence. II—Application of methods to studies of flow with circulation about a circular cylinder. Shepherd Bartnoff and Abe Gelhardt.*

1174—*The application of high-temperature strain gauges to the measurement of vibratory stresses in gas-turbine buckets. R. H. Kemp, W. C. Morgan and S. S. Manson.*

1178—*A flight investigation of the thermal performance of an air-heated propeller. John F. Darsow and James Selna.*

1196—*Investigation of effects of surface temperature and single roughness elements on boundary-layer transition. Hans W. Leipmann and Gertrude H. Fila.*

1235—*An investigation of a thermal ice-prevention system for a cargo airplane. VIII—Metallurgical examination of the wing-leading-edge structure after 255 hours of flight operation of the thermal system. Maxwell Harris and Bernard A. Schlaff.*

1274—*Compressive-strength comparisons of panels having aluminium-alloy sheet and stiffeners with panels having magnesium-alloy sheet and aluminium-alloy stiffeners. Norris F. Dow, W. A. Hickman and H. L. McCracken.*

1271—*Axial-flow fan and compressor blade design data at* 52 *degrees stagger and further verification of cascade data by rotor tests. S. M. Bogdonoff and Eugene E. Hess.*

1269—*Method for calculating wing characteristics by lifting-line theory using non-linear section lift data. James C. Sivells and R. H. Neely.*

1267—*Flight tests of a helicopter in autorotation including a comparison with theory. Alfred Gassow and Garry C. Myers, Jr.*

1266—*Flight measurements of helicopter blade motion with a comparison between theoretical and experimental results. Garry C. Myers, Jr.*

1253—*Comparison of design specifications with the actual static transverse stability of* 25 *seaplanes. Arthur W. Carter.*

1251—*An analysis of the compressive strength of honeycomb cores for sandwich constructions. Charles B. Norris.*

1250—*Effect of blade stalling on the efficiency of a helicopter rotor as measured in flight. F. B. Gustafsson and Alfred Gassow.*

1248—*Wind-tunnel investigation of control-surface characteristics of plain and balanced flaps with several trailing-edge angles of an N.A.C.A.*0009 *tapered semispan wing. H. Page Hoggard and E. G. McKinney.*

1246—*Wind-tunnel investigation of icing of an engine cooling-fan installation. James P. Lewis.*

1239—*Wind-tunnel investigation of the effect of power and flaps on the static longitudinal stability characteristics of a single engine low-wing airplane model. Arthur R. Wallace, Peter F. Rossi and E. G. Wells.*

1236—*Drag tests of an N.A.C.A.*65 (215)-114, $a=1.0$ *practical-construction airfoil section equipped with a* 0.295*-airfoil-chord slotted flap. John H. Quinn.*

The following reports have also been received:—

British Intelligence Objectives Sub-Committee

B.I.O.S. Final Reports No. 1259, 1337 item Nos. 25 and 29, 1322 item No. 9, 1345 item No. 9, 301 item Nos. 19, 21, 26, 1360 item No. 19.

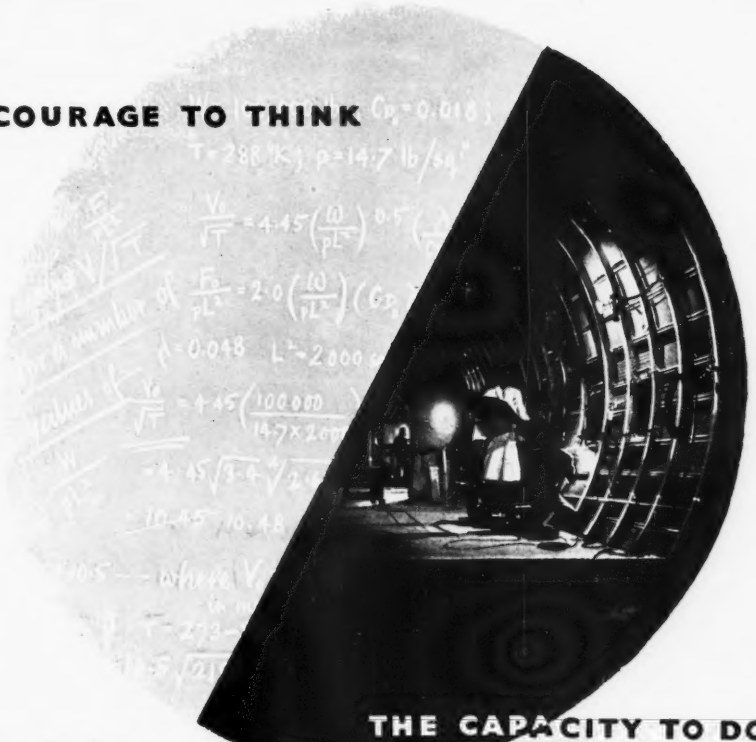
F.I.A.T. Final Report Nos. 1087 and 629.

B.I.O.S./J.A.P./P.R. Nos. 1479, 1480, 1481, 1483, 1487, 1471, 1492, 833.

J. LAURENCE PRITCHARD,
Secretary.

Made and Printed in Great Britain by the Lewes Press, Lewes, Sussex

THE COURAGE TO THINK



THE CAPACITY TO DO

Rôle of the

HAWKER SIDDELEY GROUP

As research grows more and more important in aviation, so does it become more and more difficult.
The idea behind the Hawker Siddeley Group is to make available to each member-company a variety and scope of technical experience, and resources, far beyond the practicable horizon of any company as an individual. Each company manages its own affairs; yet together they form a co-operative team which not only gives to each, in world markets, the strength of the whole, but provides also a pool of aviation technique, resources, and facilities, that to-day is unique.

A. V. ROE & CO. LTD • HAWKER AIRCRAFT LTD • GLOSTER AIRCRAFT CO. LTD
SIR W. G. ARMSTRONG WHITWORTH AIRCRAFT LTD • ARMSTRONG SIDDELEY MOTORS LTD
A. V. ROE CANADA LTD • AIR SERVICE TRAINING LTD • A. W. HAWKSLEY LTD

HAWKER SIDDELEY AIRCRAFT CO. LTD., 18 ST. JAMES' SQUARE, LONDON, SW1

Exact information, responsible commentaries and topical illustrations are among the features which have given these four representative Temple Press Publications positions of authority in the world of aviation.

TEMPLE PRESS LIMITED

BOWLING GREEN LANE, LONDON, E.C.1. TERMINUS 3636

TEMPLE PRESS

The Decca Navigator Co. Ltd.

invite you to visit their **Stand No. 107**

in the **Static Exhibition Section**

of **S.B.A.C. Flying Display 1947**

at **Radlett, Herts., September 9th to 13th**

where ***The Decca Navigator***

will be available for your inspection

The Decca Navigator Company, Limited

1-3 BRIXTON ROAD, LONDON, S.W.9 *Telephone*: Reliance 3311 *Cables*: Decnav London

WORLD WIDE AUTHORITIES

COVERING EVERY PHASE OF DESIGN, OPERATION AND PRODUCTION OF ALL TYPES OF AIRCRAFT

FLIGHT (weekly) provides the most authentic information on world aeronautical affairs. It covers every aspect of development and progress in aircraft and power unit design and operation. AIRCRAFT PRODUCTION (monthly) is the journal of the aircraft manufacturing industry, specialising in tools and works production processes. Both journals serve the interests of all concerned with the future progress of British aviation. Technical information is supplemented by brilliant functional drawings. Circulation is world wide. Annual subscriptions (Home and Overseas): FLIGHT, £3 1s. 0d.; AIRCRAFT PRODUCTION, £1 14s. 6d.

Published in conjunction with these journals, FLIGHT HANDBOOK (212 pages, 7s. 6d. net) is essentially a manual for the student, whilst GAS TURBINES AND JET PROPULSION (272 pages, 12s. 6d. net) by G. Geoffrey Smith, has been widely adopted as the standard text on the subject.

ASSOCIATED ILIFFE PUBLICATIONS

DORSET HOUSE, STAMFORD STREET, LONDON, S.E.1. WATERLOO 3333 (50 LINES)

SILICONES

Silicone products manufactured by the Dow Corning Corporation are now available in this country. They include:

D.C. SILICONE FLUIDS
D.C. SILICONE RESINS
D.C. SILICONE GREASES
D.C. SILASTIC RUBBER

Stocks are sufficient to meet demands for sample and trial purposes. Further supplies are on the way.

Enquiries for D.C. stopcock greases and Silicone fluids for high vacuum diffusion pumps should be addressed to Messrs. W. Edwards & Co., Kangley Bridge Road, Lower Sydenham, S.E.26, who are the sole distributors of these particular products.

For all other Silicone compounds please apply to

ALBRIGHT & WILSON LTD

Distributors of Dow Corning Silicones

49 PARK LANE, LONDON, W.1

Telephone: Grosvenor 1311

BW/251

STRENGTH & LIGHTNESS!

The
GOLDEN PLOVER
2,000 MILES
— NON-STOP
FLIGHT

With the first chill of approaching winter, flocks of the Pacific Golden Plover may be seen leaving the Aleutians to make landfall 40 hours later on Hawaii—an incredible non-stop flight of 2,000 miles.

The pioneering of light metals by James Booth & Company Limited in almost every development in aviation in this country—from the days of the first airships to the present time—has played no small part in enabling man to emulate by mechanical means the feat of the Golden Plover.

Our experience is at your disposal for the asking.

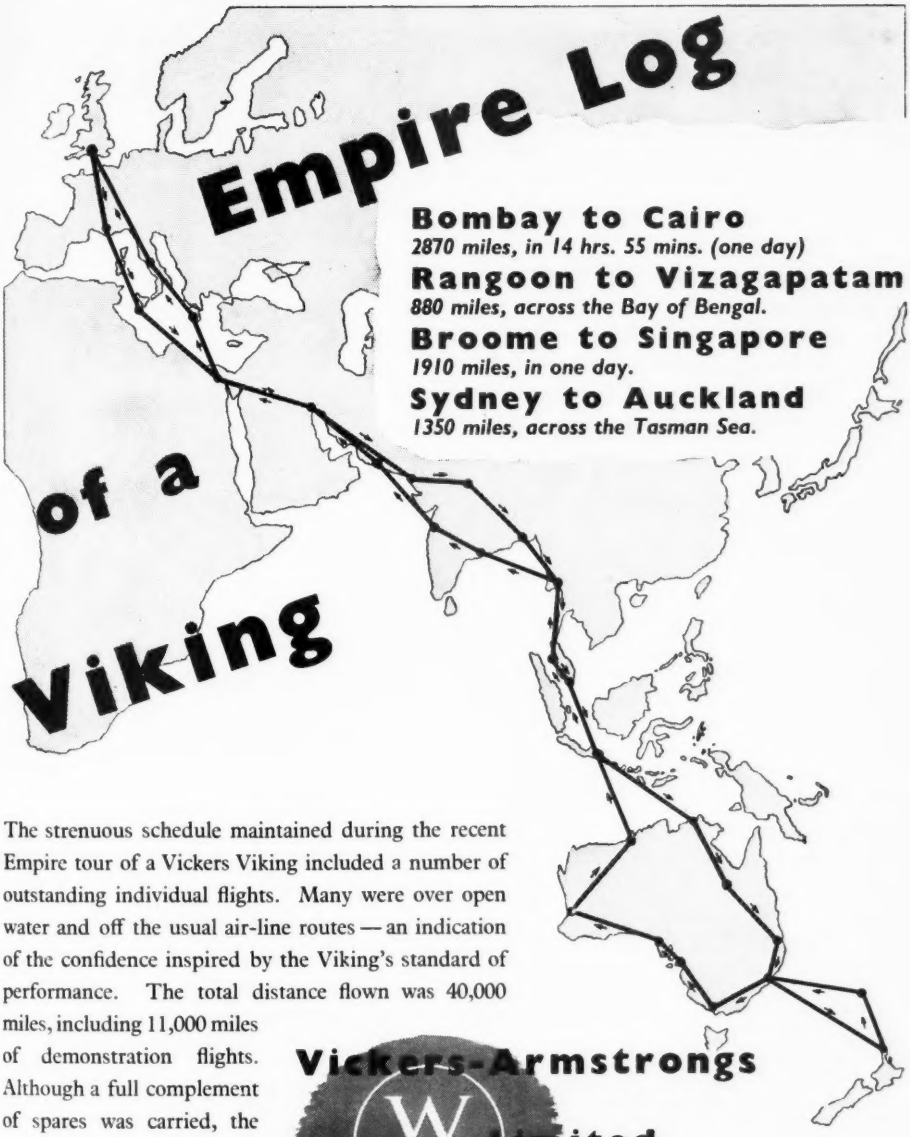
BOOTH
BIRMINGHAM

DURALUMIN · ALDURAL · DURAL · MG7

Registered Trade Marks

JAMES BOOTH & COMPANY LTD., ARGYLE ST. WORKS, BIRMINGHAM 7

Empire Log of a Viking



Bombay to Cairo
2870 miles, in 14 hrs. 55 mins. (one day)

Rangoon to Vizagapatam
880 miles, across the Bay of Bengal.

Broome to Singapore
1910 miles, in one day.

Sydney to Auckland
1350 miles, across the Tasman Sea.

The strenuous schedule maintained during the recent Empire tour of a Vickers Viking included a number of outstanding individual flights. Many were over open water and off the usual air-line routes — an indication of the confidence inspired by the Viking's standard of performance. The total distance flown was 40,000 miles, including 11,000 miles of demonstration flights. Although a full complement of spares was carried, the flight was trouble-free.

Vickers-Armstrongs Limited

A.T.20

AIRCRAFT SECTION, VICKERS HOUSE, BROADWAY, LONDON, S.W.1.

SHIPBUILDERS ★ ENGINEERS ★ AIRCRAFT CONSTRUCTORS

In Empire Air Development

FIRST Aerial Service in Eastern Australia . . . . . Charleville to Cloncurry. Hence 'Queensland and Northern Territory Aerial Services' Q.A.N.T.A.S. **1922**

FIRST Aircraft to be manufactured in Australia under licence from overseas . . . a DH50A, built by Qantas at Longreach **1926**

FIRST Flying Doctor service established in Australia by Qantas at Cloncurry **1928**

FIRST Official experimental Airmail between Australia and Britain, in association with Imperial Airways **1931**

FIRST Four engined Plane used in Australia . . . . . by Qantas on Brisbane-Singapore service (a DH86). **1935**

FIRST Through Flying Boat service to Singapore by Qantas, and — in association with Imperial Airways — to London . . **1938**

FIRST Regular Crossing of the Indian Ocean . . . the world's longest air hop . . . by Qantas in association with B.O.A.C. with Catalinas **1943**

FIRST With B.O.A.C. and Tasman Empire Airways, to re-establish the British Air Route . . . to help maintain British Empire Air supremacy in the revolutionary developments in the Age of Flight **1946**

AUSTRALIA'S INTERNATIONAL AIRLINE

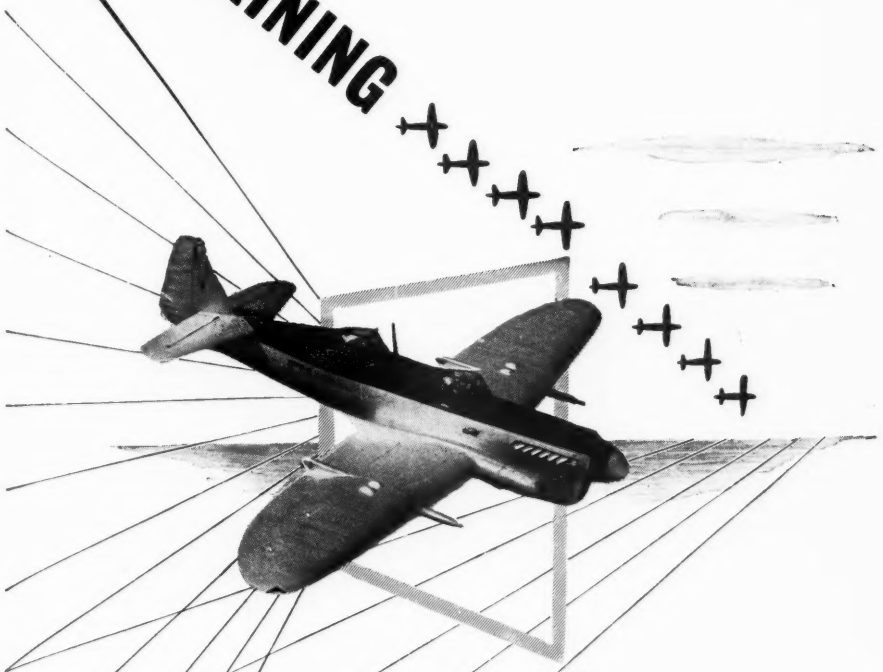
Qantas Empire Airways

in association with

BRITISH OVERSEAS AIRWAYS CORPORATION B·O·A·C

FIRST-LINE SQUADRON OPERATIONAL

TRAINING



THE FAIREY OPERATIONAL TRAINER.

the Sure Foundation of a Fighting Service

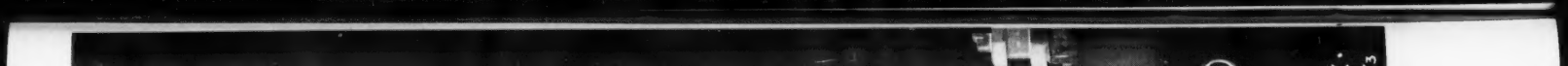
The Fairey Trainer is unique in its new class—for tactical training at high speeds and under combat conditions.

FAIREY

THE FAIREY AVIATION COMPANY LIMITED, HAYES, MIDDLESEX

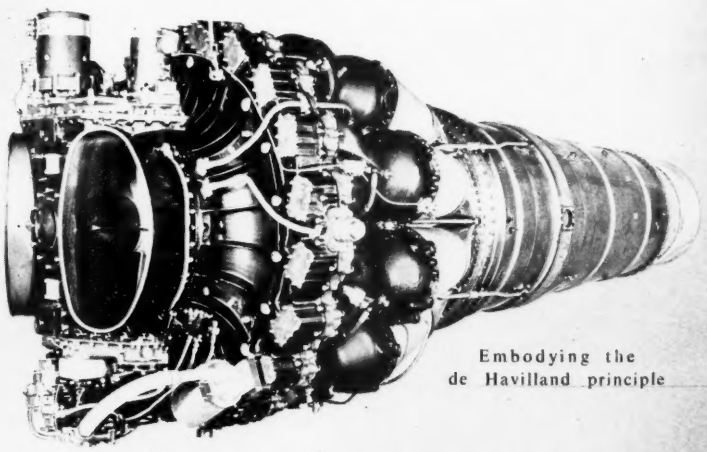
London Office: 24 Bruton Street, London, W.1. Mayfair 8791.

SKYHI
HYDRAULIC
Aircraft
SERVICE and SALVAGE
EQUIPMENT
will be displayed
for your inspection on
STAND No. 125
S·B·A·C STATIC EXHIBITION
Radlett Aerodrome Sept. 8–14
A cordial welcome awaits you
on our Stand—be sure you
pay us a visit.
SKYFLEX
'SKYHI' HYDRAULIC
AIRCRAFT JACKS
"SKYHI" LIMITED
'SKYHI' WORKS·WORTON ROAD·ISLEWORTH·MIDDLESEX·
Telegrams: SKYHIJACK · PHONE · LONDON · Telephone: HOUNSLOW · 2211/2/3



G H O S T

First gas turbine to be chosen for a jet airliner



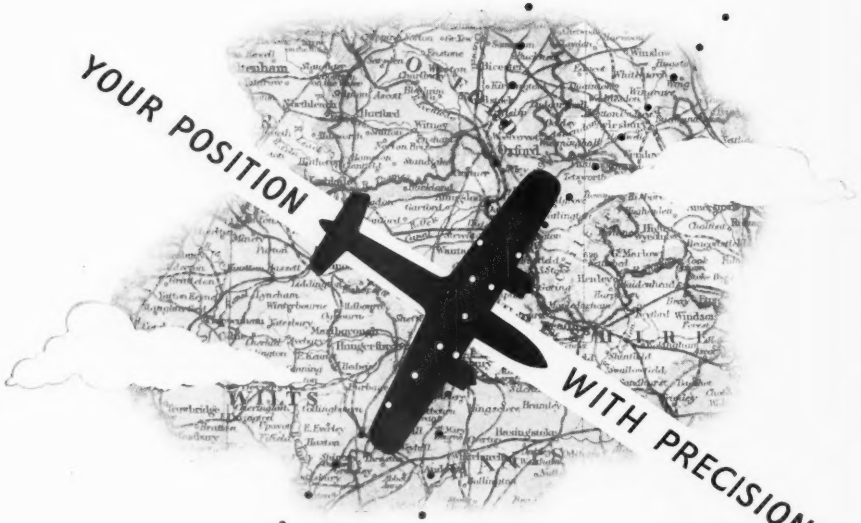
Embodying the de Havilland principle

The best on all counts

High air-intake efficiency
Low aircraft drag
Low installed weight
Low fuel consumption
Maximum accessibility
Low fire risk

D E H A V I L L A N D
J E T E N G I N E S

D H ENGINES



The story of the development of the GEE system of navigation and its vital contribution to the accuracy of aerial warfare is now a chapter in history, but its part in the future of flying will be of no less importance. It is with this knowledge that the Cossor Radar Organisation which pioneered GEE is forging ahead with new and improved equipment.

The Pulse System of Hyperbolic Navigation

GEE

PIONEERED BY **COSSOR**

Enquiries regarding GEE should be addressed to:—
Cossor Radar Ltd., Highbury, London, N.5.

CG1

DESIGNED for a decade of service, the Ambassador has been planned expressly for use on the inter-city, medium-length stages which will always carry the bulk of the world's air traffic.

Larger than aircraft at present being used over such stages, yet smaller than those for long-distance or trans-oceanic flying, the Ambassador carries up to forty-eight passengers in comfort and safety at a high cruising speed.

By refinement of design an excellent performance has been obtained for a low expenditure of power, so that the Ambassador is an economically attractive proposition. A very wide range of weak-mixture cruising speeds permits schedules to be maintained in all conditions. A high power-weight ratio offers an unusually good single-engine performance.

There is nothing "interim" about the Ambassador. Designed for to-day and to-morrow, its modern features include a pressurized and air-conditioned fuselage; a high wing for easy loading, thermal anticing; braking propellers; a steerable tricycle undercarriage with duplex wheels; integral tanks; and completely self-contained and interchangeable power plants.

Looking further into the future, the Ambassador has been designed so that it can take propeller-turbine units when these are available for commercial use.

AIRSPEED *Ambassador*

28-48 Seater

AIRSPEED

AIRSPEED LIMITED, CHRISTCHURCH & PORTSMOUTH, HAMPSHIRE, ENGLAND
ASSOCIATED COMPANIES IN AUSTRALIA, CANADA, INDIA, AFRICA & NEW ZEALAND

Two Outstanding Books . . .

. . from Pitman's

TROPICAL & EQUATORIAL METEOROLOGY

By Maurice A. Garbell. The subject of weather analysis has hitherto only been covered by statistical and climatological studies. In this book much new material, both theoretical and observational, has been introduced, including the very latest concepts and findings. It contains many excellent charts and photographs. 60s. net.

Engineering Applications of AERIAL & TERRESTRIAL PHOTOGRAMMETRY

By B. B. Talley. An expert American writer presents the subject in a manner most useful to advanced students and practising surveyors and engineers. He explains each step necessary to produce a finished map or mosaic, and deals fully with the many practical applications of photogrammetry. Well illustrated. 40s. net.

PITMAN BOOKS

. . Parker Street · Kingsway · London WC2 . .

THE AIRSPEED AMBASSADOR — aptly named for its future rôle of carrying British air prestige across the continents of the world — uses Exide exclusively. A battery that couples lighter weight with greater capacity and better performance with longer life, is the natural choice for modern aircraft.

The following aircraft are fitted with EXIDE *Batteries as initial equipment:*

AIRSPEED AMBASSADOR	MILES AEROVAN
AIRSPEED CONSUL	MILES GEMINI
AVRO TUDOR I	MILES MARATHON
AVRO TUDOR II	MILES MESSENGER
BRISTOL FREIGHTER	PERCIVAL PRINCE
BRISTOL WAYFARER	PERCIVAL PROCTOR
CUNLIFFE-OWEN CONCORDIA	SHORT SANDRINGHAM 'V'
HANDLEY PAGE HALTON	SHORT SOLENT
HANDLEY PAGE HERMES	SHORT SEALAND
VICKERS VIKING	

A9B

Exide AIRCRAFT BATTERIES

THE CHLORIDE ELECTRICAL STORAGE COMPANY LIMITED, EXIDE WORKS, CLIFTON JUNCTION, NR. MANCHESTER

B.I. Callender's

Stand No 55

S·B·A·C EXHIBITION RADLETT

We manufacture all types of electric cables including aircraft cables and radio frequency cables. Also moulded connectors, flexible waveguides for radar, aluminium sheets and sections, solder and soldering fluxes and Callender-Hamilton aircraft hangars.

All the above items will be shown on Stand No. 55 at the Static Exhibition, Radlett.

BRITISH INSULATED CALLENDER'S CABLES LIMITED

NORFOLK HOUSE. NORFOLK STREET, LONDON W.C.2

The Steels behind the
Worlds finest aero-engines
FIRTH BROWN
THOS FIRTH & JOHN BROWN LTD SHEFFIELD

THE JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY

With which is incorporated The Institution of Aeronautical Engineers

CONTENTS *SEPTEMBER 1947*

THE BEHAVIOUR OF STRUCTURES UNDER REPEATED LOADS
By A. G. Pugsley, O.B.E., D.Sc., F.R.Ae.S. :: :: 715—720

GUSTS
By W. Tye, B.Sc., F.R.Ae.S. :: :: :: :: 721—732

HIGH SPEED AIRCRAFT FLYING LIMITATIONS AND HANDLING PROBLEMS
By F. W. Morgan, B.Sc., A.R.Ae.S. :: :: :: 733—738

THE USE OF RHEO-ELECTRICAL ANALOGIES IN CERTAIN AERODYNAMICAL PROBLEMS
By L. Malavard :: :: :: :: :: 739—756

THE GENERAL THEORY OF CYLINDRICAL AND CONICAL TUBES UNDER TORSION AND BENDING LOADS, PART V
By J. Hadji-Argyris, D.E., A.F.R.Ae.S. and P. C. Dunne, B.Sc., A.F.R.Ae.S. :: :: :: :: :: :: 757—783

The Journal is published monthly at the Offices of the Society, 4 Hamilton Place, Piccadilly, London, W1. None of the papers or paragraphs must be taken as expressing the opinion of the Council of The Royal Aeronautical Society unless such is definitely stated to be the case. Subscriptions per annum, £4 13 0 including postage: Single Numbers 7/6 or 7/9 post free. All communications for publication or Advertisements in the Journal, or on general matters affecting the Society should be addressed to—The Secretary, J. Laurence Pritchard 4 Hamilton Place, W1. The Society's Bankers are Messrs. Coutts & Co.

Telephone: Grosvenor 3515 (5 lines) Telegraphic Address: Didaskolas, Audley, London.

Service of Supply

FOR JET ENGINES

DUAL

DOWTY

Fuel Pumps

DUPLEX

SINGLE

Dowty Fuel Pumps deliver 600 to 1,320 Imperial gallons per hour at 3,500 r.p.m. and 1,000 lbs. per square inch pressure

DOWTY EQUIPMENT LIMITED, CHELTENHAM, ENGLAND

THE ROYAL AERONAUTICAL SOCIETY

Paper received July 1947

THE BEHAVIOUR OF STRUCTURES UNDER REPEATED LOADS

by

A. G. PUGSLEY, O.B.E., D.Sc., F.R.Ae.S.

Professor of Civil Engineering at the University of Bristol.

IN RECENT YEARS some types of aircraft have shown signs of structural trouble due to repeated loading and exploratory thought has therefore been given to the general problems arising.[1, 2] There are two main aspects to be considered: the nature and magnitude of the loads repeatedly occurring in flight and the behaviour and strength of aircraft structures under such loads. In this country a good deal of attention has already been given to the first side of the question, mainly by *V-g* recorder work, but also by some strain gauge investigations, but the other side of the matter—the resistance of aircraft structures to repeated loading—has so far received less attention. Direct experiments have not in this country gone much beyond exploratory work on a series of Typhoon tailplanes, described by Oaks and Townshend.[3]

In these circumstances it has seemed desirable to approach this second aspect of the matter from the standpoint of tests on materials and on structural elements incorporating stress concentrations typical of aircraft structural practice. This paper outlines a picture, built up in this way, of the behaviour to be expected of elementary structures under repeated loading.

On approaching this problem from the material end, it is at once realised that one of the factors affecting the importance of the subject now and in the future is this: during the whole period of aircraft construction in metal, and indeed during the much longer period over which metal has been used in structural work generally, the trend of design has been towards the use of stronger and stronger materials (as measured by static tests) whose fatigue strength has remained relatively unchanged. This trend, together with the fact that the increasing size of aeroplanes has been rendered possible largely by reduced load factors, means, for example, that large civil aeroplanes are flying, or are about to fly, with wing stresses fluctuating about relatively high mean values, the wings being made of stronger materials than heretofore without any concomitant improvement in repeated load properties; yet it is just these aeroplanes that, for commercial reasons, are immediately required to have a longer working life!

The repeated load properties of a material in simple tension, based upon tests on polished specimens under direct tension, and envisaging endurance limits for various fixed numbers of cycles, may conveniently be displayed by a modified form of Goodman diagram. In general, even if the number of repetitions to fracture is large for given stress conditions, no limiting stress of the type envisaged by the original Goodman diagram —*i.e.* a stress below which innumerable repetitions could not produce fracture—will be found, although such stress limits do exist for some ferrous metals. The upper boundary of the Goodman diagram, such as A′B in Fig. 1, may therefore commonly be replaced by a series of boundaries, such as A_1B, A_2B, . . . , each corresponding to a given number of cycles.

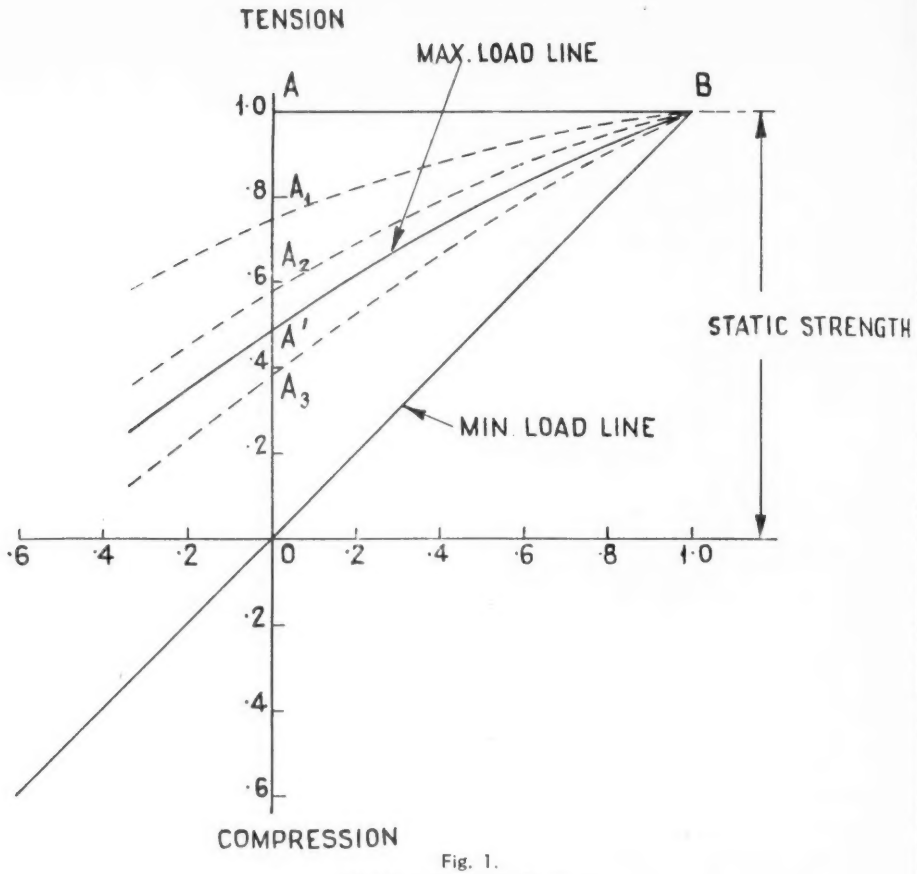


Fig. 1.
Modified Goodman Diagram.

Figure 2 shows such a diagram for a high strength aluminium alloy (27 ST, 4.5 per cent. Cu, 0.8 per cent. Si, 0.8 Mn). The curves are based upon experimental results obtained by the Aluminium Company of America[4], extrapolated to conform to the observed average static strength of the alloy concerned. The significance of these curves, and of the picture as a whole, will be realised from consideration of a sample point P on the vertical line PQR. This point indicates that a polished bar of the material, if tested under repeated loading corresponding to a tensile stress variation from a minimum value QR to a maximum value PR (range PQ), all expressible as fractions of the static strength OA, would fail at 10^5 repetitions of this stress cycle.

Suppose now we try to redraw this diagram, not for a plain tension specimen, but for some structural element more typical of structures as a whole. We observe at once that we here introduce new factors potent in fatigue effects, such as stress concentrations, and must adopt some standard in such matters as a basis for a typical diagram. Now it is common knowledge that while in general structural work it is difficult to avoid stress concentrations of the order of 4 (*i.e.* local peak stresses equal to about four times

the "average" stresses estimated by engineering theory), the fatigue effects of such concentrations, for most metals used in engineering structures, approximate to an effective concentration of roughly 2. We could then start to construct our diagram for a typical structural element by halving the ordinates to each of the points A_1, A_2 . . . , and redrawing the curves A_1B, A_2B . . . , to the general shape of those in Fig. 2. This would lead to Fig. 3.

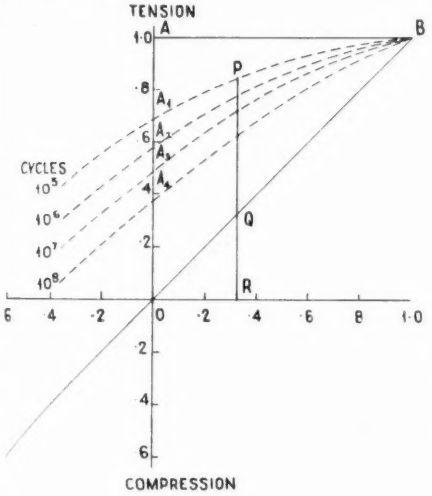


Fig. 2.
Diagram for fatigue of 27ST specimens (polished specimens in direct tension and compression).

Alternatively and better, we could examine test results for a series of specimens of typical structural elements, such as riveted and welded joints in tension, or built-up beams, taking care to concentrate on specimens representative of average engineering skill as regards resistance to repeated loads, and so begin to plot a picture similar to Fig. 3 based on direct test evidence.

Using the two alternative processes we have just described, and combining the results to construct our diagram as well as possible in spite of the meagre experimental information available, we have obtained the diagram of Fig. 4 for high grade aluminium

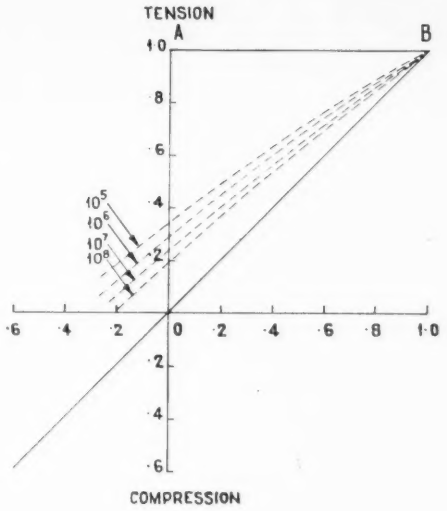
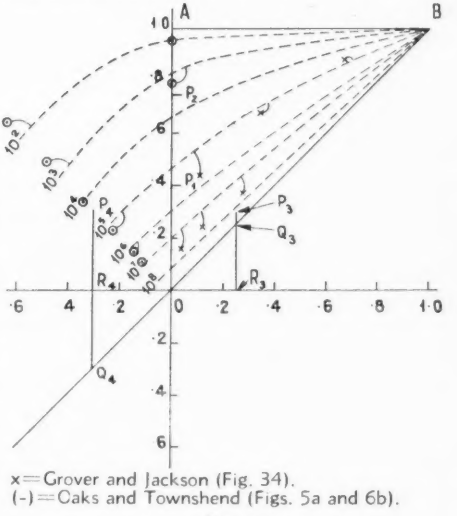


Fig. 3.
First stage towards diagram for elementary structures (high strength aluminium alloy).

alloy structures of elementary form (statically determinate, for example). As typical of this diagram in relation to direct experimental evidence, we have shown on Fig. 4 points corresponding to test results on spot welded lap joints by Grove and Jackson.[5] It will



x = Grover and Jackson (Fig. 34).
(-) = Oaks and Townshend (Figs. 5a and 6b).

Fig. 4.
Diagram for elementary structures (high strength aluminium alloys).

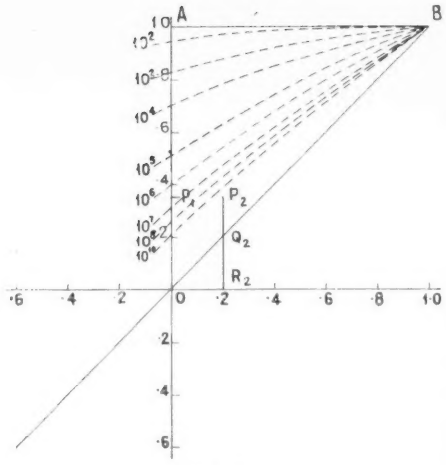


Fig. 5.
Diagram for elementary structures
(structural steel—rough approx. only).

be seen that these points justify the general form of the diagram. The same procedure, applied to structural elements made of the mild steel used in general construction, leads (although little attention has been paid to its detailed accuracy) to the diagram of Fig. 5 for structural steel. Some references to recent representative tests used for the construction of these approximate pictures are given at the end of this paper. [3, 5, 6, 7, 8, 9, 10]

Before examining these diagrams in relation to the response of structures to repeated loading, it is as well to realise that, even if the endurance lines such as A_1B, A_2B . . . were precisely supported by experiment, they could only be so supported in the sense that they represented *average* results for a typical structural component. In fact, for a series of examples of the same component, the number of repetitions to produce failure under given stress conditions would vary from one specimen to another. This is illustrated by the frequency curves of Fig. 6, which is based on some results given by Freudenthal[10] for butt welded joints in mild steel bars. Our curves in Figs. 4 and 5 can thus only indicate average trends.

Before returning to Fig. 4, which is of prime interest aeronautically, it is of interest to look at Fig. 5 in relation to general engineering practice, where experience is necessarily more extensive. Consider the two main components of a railway bridge—the principal trusses or girders, and the cross girders supporting the floor. A cross girder is only lightly loaded under the floor weight but is highly loaded every time a locomotive axle or coach bogie passes over it. As a result a cross girder on a main line railway

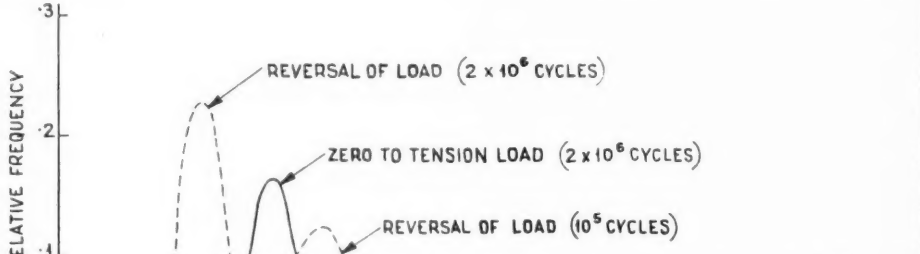


Fig. 6.
Frequency distribution of fatigue strength
(based on Freudenthal, Fig. 4, for butt welded joints in structural steel).

bridge may go through a cycle corresponding approximately to zero to a maximum load of $0.35W$ (W = static strength) for up to half a million times per year. This cycle corresponds to the line OP_1 on Fig. 5, and our rough endurance curves there suggest a life of rather under 10^7 cycles—*i.e.* of perhaps 50 years. On the other hand, the main girders of such a bridge are only loaded severely when a whole train is passing over the span, corresponding perhaps to a cycle of from 0.2 to $0.35W$ for some 20,000 times a year. For this cycle our diagram (see $P_2Q_2R_2$) suggests a life of over 10^{10} cycles *i.e.* of roughly 10^5 years. It is thus obvious that in practice the cross girders are the more likely to suffer from repeated loading troubles, and this is already being borne out by occasional partial failures of such girders on British railways, calling for their replacement. On account of the long periods involved corrosion, of course, is also playing a part in this matter.

Having thus looked at Fig. 5 in a trial way in relation to general engineering experience, we can look at Fig. 4 for aircraft structures in terms of practical loading conditions with a view to bringing out points likely to be of importance in structural design. Suppose we consider first the fact that on every flight of a given aeroplane—say a large civil aeroplane—a maximum accelerometer reading of 1.7g. is likely to be recorded. Since the specified ultimate factor for such an aeroplane corresponds to about 3.75g., the corresponding peak load on the wings might be about 0.4 of their static strength, depending mainly on the fuel load carried in the wings. We might thus regard every flight as providing a cycle of loading from roughly zero to 0.4 times the wing static strength. If we look at Fig. 4 in relation to this cycle, the loading envisaged corresponds to the line OP_1 shown and the indications are that it would be unsafe to contemplate more than about 10^5 flights for the given aeroplane, corresponding to a life of 10^6 flying hours for 10 hour flights. We must remember, however, that current design load factors correspond approximately to the occurrence of a peak load of about 0.75 of the static strength of the wings once in roughly 1,000 flying hours, and we should check this condition also. This corresponds to the line OP_2 in Fig. 4, indicating the likelihood of failure after rather over 10^3 cycles. This suggests again a flying life of at least 10^6 hours.

The above cases refer to relatively infrequent highly stressed conditions. There is, however, the fact that many aeroplane wings are executing small amplitude flexural vibrations during most of their flying life. As has been discussed elsewhere,[1] these may involve a ± 10 per cent. variation of the loading appropriate to level flight—for our example, say a load variation from 0.25 to 0.30 of the wing static strength. On Fig. 4 this corresponds to the line $P_3Q_3R_3$, indicating a life of roughly 10^8 cycles. If the vibrations occur at a frequency of about 3 per sec. for roughly 75 per cent. of each flight, *i.e.* at the rate of about 10^4 cycles per hour, then the flying life of the aeroplane will be of the order of 10^4 hours. This is of the same order as the life contemplated by civil operators, and is much less than for the previous conditions considered. If we remember that during such a period there may also occur a number of applications of peak loads ranging up to 0.75 times the static strength, then it is obvious that considerable attention should be given to the behaviour of aircraft structures under such fluctuations of loading.

A case to which attention has also been called is the reversal of loading frequently suffered by tailplanes. Such reversals may occur a number of times during each flight. If the peak loads in opposite directions are each of the order of 0.3 of the tailplane static strength, our Fig. 4, by the line $P_4R_4Q_4$, suggests a life of only 10^4 cycles, so that for a flying life of 10^4 hours such reversals should not occur more than once per hour. It is again obvious that this condition (particularly

as tailplanes also suffer flexural vibrations) is one requiring practical attention. The R.A.E. tests on the Typhoon tailplane mainly related to this case and their results are plotted on Fig. 4 to illustrate its approximate representativeness in the load reversal region.

Our picture of the behaviour of elementary structures, typified by Fig. 4, although obviously open to much detailed improvement or specialisation, thus already appears complete enough to draw a few practical conclusions:

(1) The more or less continuous vibrations of wings and tailplanes during flight appear likely to have sufficiently serious effects on conventional structures to set a period to the life of an aeroplane that is not remote from the periods contemplated commercially.

(2) The wide fluctuations and reversals of tailplane load that occur in flight may have effects on aeroplane life approaching in severity those of (1) above.

On account of these tentative conclusions, experimental work directed both to the better establishment of diagrams of the type of Fig. 4 and to the better understanding of the relative merits of different types of construction in relation to the actions (1) and (2) above, is urged. It may be that further examination of aircraft components in relation to our picture will bring forward further cases needing attention; an examination, for example, of undercarriage structures in relation to both vertical and drag loads, appears desirable.

REFERENCES.

1. A. G. Pugsley. The specification of test loads (Aircraft Engineering, Dec., 1945).
2. A. G. Pugsley. Repeated loading on structures (Symposium on the failure of metals by fatigue, Melbourne, Dec., 1946).
3. J. K. Oaks and P .H. Townshend. Fluctuating load tests on Typhoon tailplanes (A.R.C.9422, 1945, unpublished).
4. Aluminium Company of America. Structural aluminium handbook (1940).
5. H. J. Grover and L. R. Jackson. Fatigue tests on some spot-welded joints in aluminium alloy sheet materials (Journal of Welding Research Council, New York, April, 1947).
6. E. Hartmann and G. W. Stickley. Summary of results of tests made by Aluminium Research Laboratories of spot-welded joints and structural elements (N.A.C.A. Tech. Note No. 869, 1942).
7. W. Roth. The effect of repeated loadings on the strength of aircraft components. (A.R.C.9533, unpublished).
8. F. C. Lea. Repeated stresses on structures (The Structural Engineer. Jan., 1940).
9. W. M. Wilson. Fatigue tests of riveted joints (Civil Engineering, Amer. Soc. of Civil Engineers, 1938).
10. A. M. Freudenthal. Reflections on standard specifications for structural design (Proc. Amer. Soc. of Civil Engineers, Feb., 1947).

THE ROYAL AERONAUTICAL SOCIETY

Paper received July 1947.

GUSTS

by

W. TYE, B.Sc., F.R.Ae.S.

Mr. Tye joined the staff of the Air Registration Board in 1938 and is now the Chief Technical Officer. Prior to this, and for a period during the war, he served at the Royal Aircraft Establishment. He was deputy leader of the United Kingdom Delegation at the Airworthiness Division meetings of I.C.A.O. in 1946 and 1947.

Throughout his service with the A.R.B. Mr. Tye has been concerned with the preparation of airworthiness standards and has endeavoured to replace arbitrary requirements by realistic ones. In his paper he summarises the information available on gusts and points to the problems still to be solved.

1. INTRODUCTION.

1.1. Until recently the gust requirements laid down by the airworthiness authorities had not caused any serious difficulty in design. This was due to the fact that other loading conditions (manœuvring loads imposed by the pilot) had been more severe than the loads produced by gusts. Over the past few years, various factors have altered this position. Increase of aeroplane speeds, increase of operating altitudes, decrease of the manœuvring loads (in the case of large aeroplanes) and increase of size, have each contributed to raise the gust loads until they have become, in many cases, the main design criteria for the flight structure.

1.2. The purpose of this note is to summarise the extent of present knowledge of gusts, to draw attention to the information still required, and to suggest a logical framework for future gust requirements. The last-mentioned could not become effective until the gaps in present knowledge are filled.

2. GENERAL.

2.1. The aircraft designer's task would be simpler if the atmosphere consisted of either a stationary mass of air, or one moving with constant velocity. Instead, the air moves in a disconcertingly irregular way, the movements ranging from large scale effects associated with the weather, to small scale eddies caused, for instance, by local ground conditions.

2.2 So far as structural design is concerned, the movement of air is important only when large changes of velocity occur over relatively short distances. Such movements are described as "Gusts." One of the difficulties of the problem is that knowledge of the atmosphere is insufficient to permit a clear three-dimensional picture of the form of gusts to be drawn. In order to facilitate design, certain conventionalised pictures have been formulated.

2.3. The simplest conception—but one which is obviously artificial—is that of the sharp-edged gust. The assumption is that the air on one side of a plane is moving parallel with the plane at a uniform velocity (Fig. 1). As the aeroplane passes through

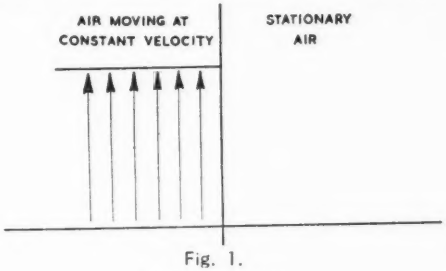


Fig. 1.
Sharp-edged gust.

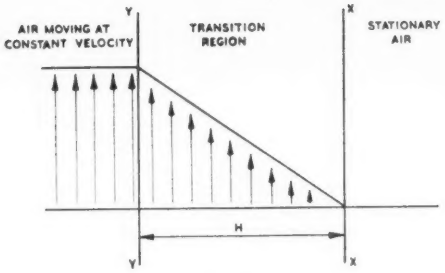


Fig. 2.
Graded gust.

the dividing plane it is subjected to a sudden change of air speed. Although this notion of a gust has merits (see paragraph 3), it is clearly improbable that nature would permit the sudden discontinuity. We should expect a transition region in which the velocity of the air increases continuously. This leads to various conceptions of graded gusts.

2.4. The simplest graded gust is illustrated in Fig. 2. In this the air is assumed to be stationary on the right of XX, and moving at constant velocity on the left of YY; between XX and YY, the velocity increases from zero to its maximum value. The rate of increase, over the gradient distance H, has been assumed by various investigators to be sinusoidal or linear.

2.5. Meteorological reports refer to "eddies" and although an eddy is not precisely described, it is possible to imagine it as a rotating cylinder of air. Proceeding from the stationary air outside the cylinder, radially inwards to the centre, one would presumably encounter first an increasing velocity, then a decrease to zero at the centre. If this conception is correct, an aeroplane flying diametrically through the cylinder would meet in turn up and down gusts with a gradient before, between, and after, the gusts.

2.6. The gusts so far visualised have been two-dimensional. Consider next the state of affairs in cumulus type clouds. Such clouds are visible indications that air is rising rapidly. We may therefore visualise a column of air rising at velocity U relative to the surrounding volume of air. Again there will be a transition region round the circumference of the column. Thus the velocity distribution would appear as a truncated cone. If the dimensions of this cone are comparable with the span of the aeroplane, it may well be that the velocity of air flow is not uniform along the span.

3. THE "EQUIVALENT SHARP-EDGED GUST" CONCEPTION.

3.1. For design purposes a simplified notion of gusts and their effects on the aeroplane has been employed. This starts with the assumption of a sharp-edged gust described in paragraph 2.3. It is then assumed that the aeroplane passes from the stationary air to the moving air in such a way that,

(a) the forward speed of the aeroplane is unaltered,
(b) the aeroplane does not pitch,
(c) the aerodynamic loads build up to the full value associated with "static" conditions (*i.e.* Wagner effects are neglected).
(d) the aeroplane is infinitely rigid.

3.2. On these assumptions, if the aeroplane encounters a gust which acts upwards or downwards, normal to the flight path, the incidence of the wings is changed to a first order of approximation by,

$$\Delta\alpha = \frac{U_E}{V_E}$$

where U_E and V_E are the gust speed and aeroplane flight speeds respectively expressed as Equivalent Air Speed.

The increment of normal acceleration is then, approximately,

$$\Delta n = \tfrac{1}{2}(\rho_o U_E V_E a)/w \quad . \quad . \quad (1)$$

where ρ_o is sea level air density
a is the slope of the wing lift/incidence curve
w is the wing loading.

3.3. The advantage of this conception lies in its simplicity in application to design. A

further advantage is that nearly all existing statistical records provide information in terms of the quantities Δn, a, V_E, and w. The sharp-edged gust thus becomes a device for translating the records obtained on one aeroplane, with its particular values of V_E, a, and w, into a design case for another aeroplane with differing values of V_E, a, and w. In the respect that these three parameters are of primary significance in determining the loads, the sharp-edged gust has its merits.

3.4. Its disadvantages arise, of course, partly from inaccuracy in the assumptions of paragraph 3.1 particularly (b), (c) and (d). If these assumptions are replaced by more accurate ones, further variables appear in the expression for Δn, and the additional variables are sometimes important.

4. MAGNITUDE OF EQUIVALENT SHARP-EDGED GUSTS.

4.1. The majority of evidence of the magnitude of gusts derives from V-g records in which the normal acceleration and associated flight speed are measured. The largest volume of records is contained in a N.A.C.A. report which gives the results obtained on a variety of civil aeroplanes covering a total flying time of 134,000 hours. The limitations of these data are:

(a) most of the flying was probably at less than 10,000 ft.,

(b) the routes covered were confined to the North American Continent, the Pacific and western Atlantic,

(c) there is no record of the associated weather conditions.

Nevertheless, the records are valuable by reason of the large number of hours of flying covered. By averaging the information for the group of aeroplanes on which measurements were taken, the conclusion is drawn that in 15,000 hours of flying the aeroplane is likely to encounter, once at least, equivalent sharp-edged gusts of the following magnitude:

34 ft. per sec. at low speeds,
32 ft. per sec. at cruising speed,
14 ft. per sec. at diving speeds.

4.2. The reason why different gust speeds are associated with various aeroplane speeds is presumably that in very rough weather the pilot tends to reduce speed; and that diving occurs infrequently so that high gust speeds are unlikely, on a probability basis, to be encountered.

4.3. In a N.A.C.A. report given restricted circulation, a large mass of American and German records are analysed. The results are presented in the form of diagrams from which it is possible to gauge the relative frequency of encountering gusts of various magnitude. These relative frequencies, expressed in terms of the frequency of a 5 ft. per sec. gust are, very approximately:

Equivalent sharp-edged gust velocity, ft. per sec.	*Relative frequency*
5	1
15	0.005
25	0.00003
35	0.0000009

The report also gives limited information regarding the number of miles of flying per gust. Estimating *very roughly* from these figures, it seems that a 35 ft. per sec. gust is likely to occur once in 1,500 hours of flying. This compares with the figure of 15,000 hours quoted in paragraph 4.1 so perhaps the fairest guess, on the basis of both sets of information, is that the gust velocities of paragraph 4.1 occur at least once in 5,000 hours of flying.

4.4. Reverting to the limitations of the above data, the first of these was the low altitude (under 10,000 ft.) at which the records were obtained. Another restricted N.A.C.A. report gives results of measuring the frequency and magnitude of gusts at several heights up to 34,000 ft., in conditions predisposing to high gust velocity. Analysis of these results indicates that, in such weather conditions, for a given frequency of occur-

rence, gust velocity varies with altitude in the following manner:

(a) from sea level to about 12,000 ft. U_E tends to increase with height,

(b) from 12,000 ft. upwards U_E is constant,

where,

U_E is equivalent sharp-edged gust velocity expressed as Equivalent Air Speed.

4.5. Above 35,000 ft. little quantitative information is available but there are certain qualitative indications, the most significant being that there is little mixing of air between the troposphere and stratosphere. This suggests that at the tropopause, gusts are of small magnitude, or if high gusts occur, then only infrequently. It would be expected therefore that above some altitude a gradual reduction of gust velocity would occur. The quantitative data previously described were obtained in or near storm clouds. There is, in addition, information to show that the frequency of storm clouds diminishes with height above about 25,000 ft. Thus it is reasonable to suppose that the frequency of occurrence of a given gust velocity reduces with height. Evidence of this kind was submitted at a recent I.C.A.O. meeting by the U.S.A. The measured frequency of occurrence of the tops of cumulus type clouds, in terms of 100 per cent. frequency at 25,000 ft. or below is:

Height ft.	*Frequency* per cent.
25,000	100
30,000	60
35,000	15
40,000	8

After "fairing" these experimental data and combining them with the tentative conclusion that in such clouds the gust speed (E.A.S.) is constant, it is possible to show that, approximately, the gust velocity on a constant frequency basis, varies as $\frac{U_E}{\sqrt{\sigma}}$ E.A.S. above 25,000 ft. This extrapolation should be carried no farther than, say, 40,000 to 50,000 ft. It will be noted that constant $\frac{U_E}{\sqrt{\sigma}}$ E.A.S. is equivalent to constant true air speed. The variations of gust speed with height are shown in Fig. 3.

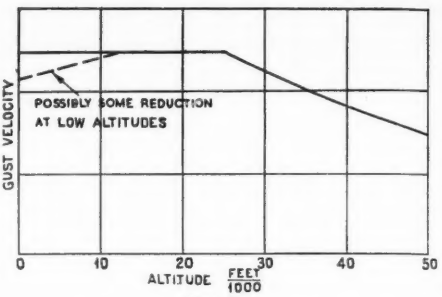


Fig. 3.

Variation of gust. Velocity with altitude.

4.6. As mentioned in paragraph 4.1, most available *V-g* recorder evidence does not differentiate between the various weather conditions in which the records were obtained. It is, however, possible to draw some conclusions. It is first necessary to define two kinds of flight conditions.

(a) Conditions in which a pilot might reasonably anticipate rough air. This includes flight in or near cumulonimbus clouds and flight at low altitudes over mountainous terrain.

(b) Conditions in which a pilot might reasonably anticipate smooth air. This includes flight remote from cumulonimbus clouds and mountainous terrain.

While it is common knowledge that in "anticipated rough air" the air is in fact often rough, it is not certain whether in "anticipated smooth air" the air is always smooth. Some accidents suggest that unexpected large gusts sometimes occur in apparently smooth air.

4.7. Turning to the flight technique which the pilot employs; in anticipated rough air, the aeroplane speed is usually reduced (usually to 1.5 to 1.7 times the stalling speed) in advance of meeting the rough air. Once the rough air is encountered, however, an

upset may occur during the recovery from which the aeroplane gains some speed. If smooth air is anticipated the aeroplane speed will usually be the cruising speed.

4.8. We may therefore conclude that:

(a) the 34 ft. per sec. gust (paragraph 4.1) at low aeroplane speeds, is very probably associated with anticipated rough air

(b) the 32 ft. per sec. gust at cruising speed may be associated either with:

(i) unexpected gusts in anticipated smooth air, or

(ii) gusts in anticipated rough air, when the aeroplane has gained speed after being upset by the rough air.

The absence of a definite conclusion to (b) is of little importance while the cruising speed and the speed achieved in an upset in rough air are of the same order. In future designs the cruising speeds may be higher than the rough air upset speed, in which case it becomes important to know whether the 32 ft. per sec. gust needs to be applied at cruising speed or whether a smaller value is acceptable.

5. THE GRADED GUST.

5.1. More recently the graded gust, described in paragraph 2.1 has been used for design purposes, both directly and indirectly. In both cases a gradient distance H and a velocity distribution within the distance H, are assumed. The "direct" method consists of making a full calculation of the motion of, and the loads applied to, the aeroplane during the entry into the gust. This method increases in accuracy according to the number of variables taken into account. It involves, however, long and complex arithmetic and usually is only worth while in the case of large aircraft where accuracy cannot otherwise be achieved.

5.2. The "indirect" method is to make generalised calculations which can be carried up to a point such that subsequent application to a particular case involves little calculation. These generalised calculations have a common aim—to determine an "alleviating factor" F, such that the loads produced by an actual gust U of gradient distance H, are given by an equivalent sharp-edged gust $U_E = FU$. Thus the calculation applied in any particular case is simply the determination of the factor F from the generalised values, followed by the calculation of the loads from the equivalent sharp-edged gust. The factor F takes account of as many of the variables as possible.

5.3. An early investigation was made by Rhode and, employing his work, various airworthiness authorities have developed simplified alleviating factors. The British Civil Airworthiness Requirements are typical. In these a linearly graded gust of distance $H = 100$ ft. is assumed. Taking account of rate of build up of aerodynamic forces and assuming average values for stability, aeroplane size and air density, and ignoring structural flexibility, a factor F is quoted as a function of wing loading, as follows:

$F = 0.3w^{\frac{1}{3}}$ where w is less than 16 lb./sq. ft.

$F = 0.8 - \frac{1.6}{w^{\frac{1}{3}}}$ where w is greater than 16 lb./sq. ft.

5.4. More recently the Netherlands National Research Institute has investigated the problem. Proper account is taken of gradient distance wing loading, aeroplane chord, air density, slope of the lift/incidence curve and stability in pitch. Flexibility is ignored. The alleviating factor is then shown in terms of three parameters in which the above variables appear in combination.

5.5 Thus at present there are two satisfactory courses open:

(a) where structural flexibility is important, to make full calculations,

(b) where structural flexibility is not important, to reduce the actual gust to its equivalent sharp-edged value by employing an alleviating factor which takes due account of the main parameters, except flexibility.

6. FLEXIBILITY OF STRUCTURE.

Flexibility of the aircraft structure is unimportant, provided that the load is applied slowly by comparison with rate at which the structure oscillates. When the rate of application of load is of the same order as one of the resonance frequencies, the effect is usually an increase of the transient stresses above the stresses which would arise if the structure were rigid. If the gradient distance is H and aeroplane speed is V, the time for the load to build up is of the order of $\frac{H}{V}$. If the lowest natural frequency of the structural part (*e.g.* the wing) corresponding to its fundamental mode, is F, the longest time for a complete cycle is $\frac{1}{F}$. Investigations by the Netherlands suggest that flexibility is not likely to be of significance when $\frac{H}{V}$ equals or exceeds 0.5 times $\frac{1}{F}$.

It is usually in large aeroplanes, in which F is low, that flexibility becomes important under gust loads. For example, if H is 100 ft. and $V=400$ f.p.s., according to the Netherlands results if F is less than 2 cycles per sec. the transient stress effect may be ignored.

7. GRADIENT DISTANCE.

7.1. To illustrate the effect of gradient distance consider a typical large aircraft of wing loading 40 lb. per sq. ft., $dc_l/d\alpha 4.5$, relative density 0.5.

The magnitude of the equivalent sharp-edged gust U_E corresponding to an actual gust $U=50$ ft. per sec. for various distances H is as follows:

$H=$	25	50	100	150	ft.
$U_E=$	0.74	0.73	0.70	0.67	ft. per sec.

The effect is much more pronounced if the wing loading is small.

7.2. Various calculations suggest that the velocity distribution is of less importance and for practical purposes it seems that either a linear distribution or a sinusoidal one would be sufficiently near the truth.

7.3. The evidence of the magnitude of the gradient distance H is scanty. In present British requirements it is assumed to be 100 ft., but this assumption is made primarily with a view to leading to results consistent with V-g evidence. Thus a gust velocity $U=50$ ft. per sec. at cruising speed is specified. Associating this with a gradient distance of 100 ft., and applying it to aircraft of the type on which the main mass of V-g evidence is available (paragraph 4.1), leads to an equivalent sharp-edged gust of 33 ft. per sec., which is broadly consistent with the evidence.

7.4. There is some additional information to support the choice of 100 ft. gradient distance. In an unpublished report by the N.A.C.A., some hundreds of results are given of velocity and gradient distance. This information suggests that as gust velocity increases gradient distance is likely to increase. Superficially it might be expected that the gust velocity T.A.S., would be directly proportional to gradient distance (*i.e.* a constant gradient). The envelope of the plotted points of the U.S.A. data suggests, very roughly, that $\frac{U}{50}=\left(\frac{H}{100}\right)^{\frac{1}{3}}$ (the envelope passing through $U=50$, $H=100$).

In a recent R.A.E. report an analysis is made of the U.S.A. information and constant probability curves are drawn, *i.e.* curves showing the gust velocity plotted against gradient distance on an equal probability of occurrence basis.

7.5. The lack of information, however, precludes the drawing of any reliable conclusions. Statistical evidence is required of gust magnitude and gradient distance in rough air and smooth air over a complete range of altitudes.

7.6. In many U.S.A. reports special significance is attached to the size of the aeroplane in determining the gradient distance. In these reports the conclusion is drawn that the aeroplane is most responsive to gusts the gradient distance of which is 9 chord lengths.

It may be that for large gradient distances, the stability of the aeroplane and the reaction of the pilot reduces the gust effect, while for short distances the aerodynamic loads do not have time to build up to full magnitude. However, the peculiar significance of 9 chord lengths is not wholly explained. Clearly the atmosphere is unaware of the size of the aeroplane flying through it and in order to approach the problem logically, a chart of the atmosphere unrelated to its effect on the aeroplane is the first essential.

8. MAGNITUDE OF ACTUAL GUSTS.

8.1. At this point it is relevant to refer again to the magnitude of the gust velocity. The results of paragraph 4.1 were expressed in terms of equivalent sharp-edged gusts. The British Civil Airworthiness Requirements assume a gradient distance of 100 ft. On this assumption the comparable actual gust velocities can be determined. For the aeroplanes for which these data were obtained an alleviating factor F would be of the order 0.66. Thus the actual gust velocities deduced from the equivalent sharp-edged gust of paragraph 4.1 become:

51 ft. per sec. at low speeds,
48 ft. per sec. at cruising speed,
21 ft. per sec. at diving speeds.

For comparison we may quote present British requirements:

66 ft. per sec. at a "rough air speed,"
50 ft. per sec. at cruising speed,
25 ft. per sec. at diving speed.

The somewhat higher magnitude of the second group of figures is associated with a more remote frequency of occurrence. There is also some additional evidence to support the choice of a 66 ft. per sec. gust velocity to be derived from meteorological sources and evidence from glider pilots. On occasions upward vertical velocities of 65 to 75 ft. per sec. in clouds have been quoted.

8.2. No distinction has yet been made between gusts in various directions, upwards, downwards and horizontally. It seems likely that eddies caused by wind gradient or by ground effects produce equal velocity gusts in all directions, but these gusts are probably of relatively small magnitude. In the case of gusts arising from convection currents, and made apparent by the associated clouds, there is some evidence that upward currents exceed the downward ones. This is reasonable if one views the upward rising column of air as having smaller cross sectional area than the downward moving air surrounding the central column. If the previously mentioned V-g recorder evidence is divided into upward and downward velocities and expressed as actual gusts for comparison with the above Table, the results are:

Gust speed, ft. per sec.

Up	*Down*	
56	45	At low speeds
48	47	At cruising speed
26	17	At diving speed.

8.3. No definite conclusion can be drawn from this as the cruising speed figures are virtually equal, while there is a considerable difference at the low and high speeds. In an unpublished U.S.A. report it was concluded that horizontal and vertical gusts are of about equal magnitude, but there is little genuine statistical evidence to support or refute it. Of the various problems this is perhaps of least importance at present, as horizontal gusts and down gusts are not so significant in determining the structure weight of the aeroplane as are up gusts.

9. LOSS OF CONTROL IN GUSTS.

9.1. In paragraph 4.5 reference was made to the common practice of reducing speed in gusty weather. Equation (1) shows that the loads due to a given gust vary linearly with the aeroplane speed. It is safer therefore to fly slowly provided that, in doing so, loss of control is not invited. Loss of control in gusty weather is potentially dangerous, since in the period of regaining control the aeroplane speed is likely to increase and a severe gust may be encountered while the aeroplane

is flying at this higher speed. The problem is therefore to determine an optimum rough air speed, so as to balance the danger of direct failure from high velocity gusts against the risk of frequent loss of control, with consequential hazard in the recovery period.

9.2. Some accident reports suggest that the danger following loss of control is a very real thing. There is, however, no simple criterion by which to measure the likelihood of loss of control. It is probable that if the aeroplane is flying very close to its static stalling speed V_s, gusts would produce frequent stalls some of which would have dangerous consequences. Investigations by the N.A.C.A. indicate that very rarely is the gust distributed absolutely symmetrically along the span. Hence it is likely that gusts tend to produce stalling of one wing and not the other, with a resulting tendency to roll. Good aileron control above and below stalling speed is thus a desirable characteristic.

9.3. I.C.A.O. Airworthiness Standards draw attention to a speed which in effect is an optimum rough air speed. This is based on the apparent desirability of preventing symmetrical stalling in the roughest air. Having specified a maximum gust speed (which in I.C.A.O. Standards is 66 ft. per sec. or in terms of equivalent sharp-edged gust speed of the order 40-45 ft. per sec.) it is possible to calculate a flight speed V_B corresponding to stalling under the specified gust. The following equations apply:

$$n = 1 + \tfrac{1}{2}(\rho_o U_E V_E a)/w \quad . \quad (2)$$

$$n = C_{Lmax}\ \tfrac{1}{2}\rho_o V_E^2/w \quad . \quad (3)$$

If these two equations are plotted on a $V - n$ basis, Fig. 4, the intersection of the curves gives the value $V_E = V_B$ at and below which the gust U_E would cause stalling. For purposes of a simple definition of an optimum rough air speed, this speed V_B appears satisfactory, in the absence of anything better, were it not for certain practical difficulties.

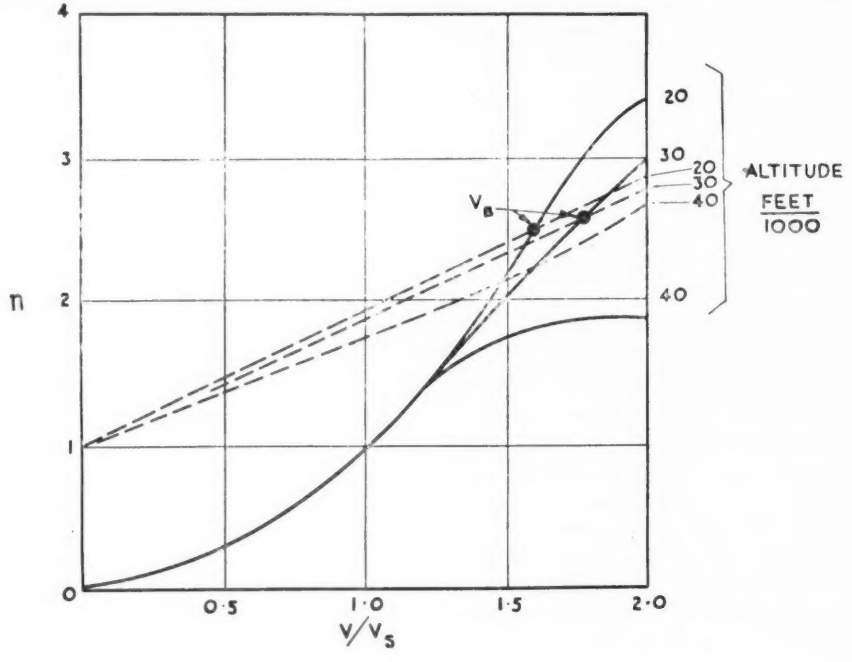


Fig. 4.
Determination of speed V_B.

9.4. The value of V_B for a typical modern aircraft works out to be about 1.6 V_s at low altitudes. The effect of high altitude, however, is to raise the Mach number, which increases a and reduces $C_{L\ max}$. The intersection of the two curves (Fig. 4) therefore moves to the right, indicating increase of V_B. At moderate heights, typical aerofoil sections lead to V_B of the order of $2.0V_s$ which may exceed the speed at which flight is possible because of power limitations (Fig. 5). At

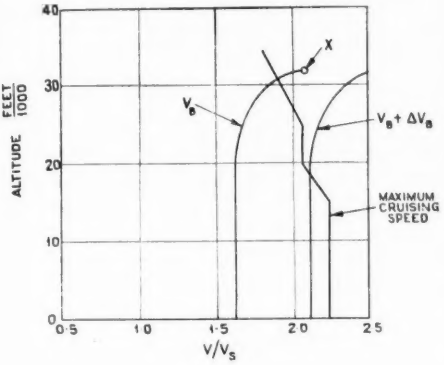


Fig. 5.
Variation of V_B with altitude.

higher altitudes the two curves of Fig. 4 may not intersect at all, which means there is no speed at which the aeroplane can fly to avoid danger of stalling in the specified gust velocity (point X on Fig. 5). Thus the difficulties of applying this particular criterion for safety against loss of control make it essential to prove its necessity before applying it.

9.5. More investigation is necessary to establish a sound basis for such a speed. One point, however, is worth noting. If it is found eventually that gusts of high velocity persist at all altitudes and if it is also found that such gusts cause dangerous loss of control when flying at low flight speeds, it may well be that the minimum safe flying speed will dictate the power required at high altitudes—alternately for a given power there may be a maximum safe altitude in rough air.

10. SPANWISE DISTRIBUTION OF GUST LOADS.

10.1. In the previous paragraphs reference was made to the asymmetric distribution of gusts along the span. If a gust consists of an eddy with a horizontal axis and the aeroplane's path through the centre of the eddy is other than normal to this axis, the gust will be asymmetrical along the span. Similarly, if a gust is a vertical column of rising air the whole span may not be included in the column, producing asymmetry. As size of aeroplanes increases, the chance increases of their being a variation of gust velocity along the span. So far as structural strength is concerned this asymmetry may not be so serious as it would appear at first sight. In aeroplanes in which most of the weight is in the fuselage, the rolling moment of inertia is so small that the likelihood of large bending loads occurring in the wings due to asymmetry is small. With very large aeroplanes this is not necessarily the case. However, in one example calculated by the Bristol Aeroplane Company, a very large aeroplane was assumed to appproach a linearly graded gust ($H=100$ ft.) in a direction of 45° to the gust front. In the case of both up and down gust the shear and bending moments were found to be greater with the symmetric than the asymmetric gust, except in the rather special case when the ailerons were considered to be operated to alleviate the gust effects.

10.2. If, as seems likely, maximum intensity gusts are associated with rapidly rising air leading to formation of cumulus clouds, the distribution of gust velocity is then probably as described in paragraph 2.6, *i.e.* a column of rising air surrounded by a circumferential belt of air forming the transition region. In such a formation the diameter of the column plus its transitional belt may be of the same order of size as the span of large aeroplanes. Particularly, one would expect that where the gradient distance was small, the overall diameter might also be

small. If this is the case, and at present the matter is one of pure speculation, it is possible that very large aeroplanes may be less affected by gusts, on the grounds that if the gradient distance is short enough to make the gust load serious, this load will not apply to the whole span of the aeroplane.

11. MISCELLANEOUS PROBLEMS.

11.1. TAILPLANES.

The loads on a tailplane when flying through normal gusts can be calculated with some precision, if the whole motion of the aircraft is determined. For simplicity it is desirable to reduce the gust to its equivalent sharp-edged value and then to proceed as in the case of the mainplanes. This in fact is the current practice. The actual gust U is multiplied by an alleviating factor F, the factor being determined by the *wing* characteristics, since it is the wing which primarily determines the response of the aeroplane. The change of incidence of the tail is then taken as $\frac{FU}{V_E}$ less an allowance for the change of the downwash as the wing enters the gust. The validity of this procedure is questionable. The effects of pitch are likely to be more pronounced on the tailplane loads. Moreover, the tail is several feet behind the wing (the tail arm often being comparable in dimensions with the gradient distance). More investigation is necessary before too much reliance can be placed in the present simplified method.

11.2. FIN AND RUDDER.

The R.A.E. reports a study of the loads on the fin and rudder in lateral gusts. Again, the aim was to make generalised calculations to determine a simple form of alleviating factor. These calculations indicate that as the aeroplane enters the gust it yaws into the gust and that both the angle of yaw and rate of yaw relieve the load on the fin and rudder.

There is also some lateral movement of the aeroplane as a whole, but the effect on fin loads appears to be small. As the body enters the gust in advance of the fin and rudder, the ratio of the destabilising moment of the body to the stabilising moment of the fin and rudder is significant. Reasonably exact estimates of the resulting loads could be made, were it not for the absence of information on the rate of build up of aerodynamic forces on fuselage shapes. Since the motion is primarily one of yawing, the significant variables are the moment of inertia in yaw and the forcing moment which is a function of fin and rudder area and aspect ratio and tail arm. The size of the aeroplane in terms of tail arm, in relation to the gradient distance H is also important.

The British Civil Airworthiness Requirements give an alleviating factor as a function of three parameters in which the above variables appear in combination. Apart from small and lightly loaded aeroplanes, the alleviating of which is of the order 0.6 to 0.8, most modern aeroplanes have alleviating factors about 1.0, the effects of yaw being small. Although these alleviating factors are, at the best, rough approximations, they are probably sufficiently exact for most practical purposes.

12. RATIONAL DESIGN REQUIREMENTS.

12.1. In suggesting a rational form of design requirements, it is supposed that there is a complete fund of information available. The first step is to consider two quite separate flight conditions:

(a) when rough air is anticipated,

(b) when rough air is not anticipated.

In separating these conditions it is supposed that any form of gust detector will be a somewhat crude device which will not be capable of detecting all air movements. On this basis we may assume that in condition (a), the pilot will restrict flight speed to a value V_B and in condition (b) to a higher speed V_C. The speed V_B must not be less than the minimum speed for adequate control in rough air. Strength must be provided for meeting the highest probable magnitudes of gust U_1.

12.2. It must then be presumed that occasionally, when flying at the selected speed V_B, the aeroplane will be upset by gusts and will acquire additional speed during recovery. The increment of speed will depend on the characteristics of the aeroplane. At this higher speed $V_B + \Delta V_B$, strength must be provided for meeting a high velocity gust U_2, but U_2 will probably be less than U_1.

12.3. Turning to condition (b), the selected speed V_C will necessarily not be less than the cruising speed of the aeroplane, since it would be desirable to limit the flight above this speed to emergencies only. Strength should be provided for a gust speed U_3, corresponding to maximum intensity in anticipated smooth air. As discussed earlier, V-g recorder evidence suggests that either U_2 or U_3, or both, should be of the order 50 ft. per sec., but it is not clear from the evidence which, if either, is the smaller. The writer's opinion, admittedly based on limited meteorological reports, is that U_3 should be smaller than U_2 perhaps having a value about 40 ft. per sec.

12.4. Finally it must be assumed that, however strict the operating limitations are on flying above speed V_C, occasionally the diving speed V_D will be reached. In large civil aircraft the dive can be taken as an inadvertent action, following, say, the loss of control at speed V_C. Hence V_D should not be less than $V_C + \Delta V_C$, where ΔV_C is the increment of speed gained in recovery. In military aircraft and small civil aircraft it may be necessary to provide higher values of V_D for intentional manœuvres. At the selected value of V_D strength must be provided for a small gust velocity U_4.

12.5. When referring to the gust speeds U_1, U_2, U_3 and U_4 above it should be remembered that as information grows, each such speed should vary appropriately,

(a) with altitude,
(b) with direction, upward, downwards and horizontally,
(c) with gradient distance (*i.e.* for each flight condition a U/H curve may be necessary).

12.6. The following Table shows the extent to which present civil requirements

TABLE

Suggested form of requirements		I.C.A.O. airworthiness standards		Comments
Aeroplane flight speed.	Gust speed.*	Aeroplane flight speed.†	Gust speed.† f.p.s. E.A.S.	
Minimum for control in rough air—V_B	U_1	V_B = speed at which 66 f.p.s. up gust stalls aeroplane or $1.6V_C$ whichever is less	$U_1 = 66$	Logical, but basis for calculation of V_B doubtful
Minimum for control plus margin for upsets—$V_B + \Delta V_B$	U_2	$V_C = V_B - 50$ m.p.h. E.A.S. but need not exceed 0.9 times maximum cruising speed	$U_2 = U_3 = 50$	Illogical, as (a) evidence to support assumption that $U_2 = U_3$ is lacking, (b) allowing V_C to fall below $V_B + 50$ may be dangerous
Normal operating speed in smooth air —V_C	U_3			
Normal operating speed plus margin for upsets—$V_C + \Delta V_C$	U_4	$V_D = V_C + 70$ m.p.h. E.A.S. but need not exceed T.V. in 30° dive	$U_4 = 25$	Logical

*With appropriate altitude variation. †Values apply to altitudes up to 20,000 ft.

(I.C.A.O. and British) approach this rational arrangement.

13. SUMMARY.

13.1. It only remains to summarise the extent of present information and the numerous deficiencies. In listing these deficiencies a personal opinion is given of the importance of filling in the various gaps.

13.2. Present information is confined to:

(a) an overall evaluation of the frequency of occurrence of gusts of various magnitudes at low altitudes in normal airline operating conditions,

(b) limited information regarding the variation of gust magnitude with altitude in conditions predisposing towards rough air,

(c) limited information regarding gradient distance in relation to gust velocity,

(d) theoretical calculations to establish the loads (in terms of alleviating factors) on aeroplanes in defined gust conditions.

13.3. The present fund of information requires amplification as follows: Items (a) and (b) are essential if the design of high speed, high altitude aeroplanes is to lead to safe, and at the same time economical, results. Item (c) is essential for large aircraft design and desirable in all cases. Items (d) to (f) are desirable aids, but not so essential.

(a) *Effects of weather and altitude.*

Statistical evidence is necessary to determine, on a frequency of occurrence basis, gust velocity variation over the complete range of heights in "smooth" air and "rough" air.

(b) *Flight speeds.*

Flight tests are necessary to establish a criterion for minimum speed for adequate control in rough air.

(c) *Gradient distance.*

In the case of large aeroplanes where flexibility is important, the sharp-edged device breaks down and the knowledge of gradient distance is necessary. Statistical measurements are necessary over a range of heights in "smooth" air and "rough" air to relate, on a frequency basis, gust velocity and gradient distance.

(d) *Speed margins.*

Statistical evidence is desirable to establish the necessary "spread" of speed to cater for loss of control.

(e) *Relative magnitude of upward, downwards, and horizontal gusts.*

Statistical evidence is desirable.

(f) *Tailplane and fin and rudder loads.*

Further theoretical development of simplified alleviating factors is desirable.

14. ACKNOWLEDGMENT.

In writing this paper I have had the advantage of reading many reports which have not yet been given general circulation. Specific acknowledgment to the authors of these reports is therefore not made, but I wish to record my appreciation of the work done by members of the N.A.C.A. at Langley Field, the Royal Aircraft Establishment and the Netherlands National Research Institute. The various opinions expressed are, of course, personal.

THE ROYAL AERONAUTICAL SOCIETY.

Paper received February 1947

HIGH SPEED AIRCRAFT FLYING LIMITATIONS AND HANDLING PROBLEMS

by

F. W. MORGAN, B.Sc., A.R.Ae.S.

Since 1939 *Mr. Morgan has edited and drafted technical recommendations for the operation of aircraft in flight for the Air Ministry and Ministry of Aircraft Production, now Ministry of Supply. For the past two years he has made a special study of the problems affecting the flying of aircraft at high Mach numbers, for the benefit of senior pilots flying high-speed types.*

PART I.

A consideration of the factors suggesting that new types of flying instruments are becoming necessary.

1. Speed limitations.

1.1. To safeguard the aircraft and crew against the danger of excessive speed and normal acceleration loads, until recently it was considered sufficient to provide for the pilot an airspeed indicator and to fix in the aircraft a plate giving the maximum safe speeds permitted for diving, without and with external stores, flaps and undercarriage down and so on. In the case of Service aircraft these limitations are usually published in Pilot's Notes for the type. Based on design data these speeds are those at which, allowing suitable factors of safety, the stresses in the appropriate parts of the aircraft's structure are calculated to reach the maximum safe limits.

1.2. In the case of fighter type aircraft designed for manœuvrability at high speeds, the maximum safe diving speed has hitherto usually been calculated for a normal acceleration loading of, say, 6g with a safety factor of two; the maximum speed being that at which calculation shows the weakest member in the structure will fail, because of the compound stresses imposed by the air loads on the structure when subjected to a normal acceleration loading of 12g.

1.3. Without a pressure suit the pilot's g threshold is considerably below 12g and he is, therefore, able to sense when the g he is putting on is becoming excessive well before it reaches the safe limit. He is able to sense this notwithstanding the fact that, as tests in the case of experienced pilots have proved, he may be quite unable to distinguish, by the sense of pressure on his seat, between a loading due to a normal acceleration of, say, 3g and one due to 5g. The precautions outlined above, therefore, have proved quite adequate until recently.

2. Guarding against compressibility effects.

2.1. The advent of aircraft capable of reaching speeds, even in level flight, at which compressibility effects are encountered has introduced new factors necessitating additional precautions, and the necessity for other types of instruments, in addition to (or perhaps instead of) the usual airspeed indicator, is becoming increasingly apparent.

2.2. The behaviour of aircraft at Mach numbers at which compressibility effects become apparent is already beginning to be known. At the critical Mach number a noticeable change of longitudinal trim occurs, usually to nose down, often accompanied by buffeting, and the nose or either wing may drop as shock waves build up and a stall develops. In addition to the pure com-

pressibility effects, an initial change of trim may occur before the critical Mach number is reached. This initial trim change, which may have either a nose down or nose up tendency, is primarily due to distortions of the airframe, particularly of the tail surfaces, and is more marked at low altitudes where the air is denser. Such trim changes should not be confused with true compressibility effects the onset of which, however, may be encouraged thereby. In any case, the true compressibility trim changes will supervene as the critical Mach number is reached.

2.3. The speed of sound decreases as the air temperature falls and the Mach number at which compressibility effects are encountered consequently occurs at decreasing true (and even more rapidly decreasing indicated) air speeds as height is gained—at least up to the tropopause, at about 36,000 ft. under average conditions.* This involves imposing a sliding scale of maximum permissible indicated air speeds decreasing with altitude. Speed limitations based on such a sliding scale have already been issued for several types now in service in the R.A.F. or Navy. The observance of such limitations imposes upon the pilot an effort of memory (or the necessity for constant reference to a data plate), with consequent possibility of fatal error, which is clearly undesirable.

To meet this an instrument known as a Mach meter, which indicates the Mach number of flight at any height, has already been developed and is now being fitted in certain high-speed aircraft. With this instrument, the pilot merely requires to know the limiting Mach number (this will usually be slightly below the critical Mach number at which local shock stalling begins to occur, thus allowing for the effects of bumps, distortion of the airframe and so on) for the aircraft, which of course remains substantially constant at all heights and, since he has but one figure to remember and one instrument to watch, the possibility of exceeding the limit is greatly reduced.

Should it prove necessary to impose an over-riding indicated air speed limit effective at low altitude, the retention of a normal airspeed indicator will be necessary (this may be desirable for other reasons also—see para. 3.7). To meet this requirement a combined airspeed indicator and Mach meter is now being developed.

3. The effects of normal acceleration —g at high Mach numbers.

3.1. It may be argued that aircraft should be designed to withstand the total loading and changes in distribution of load on the structure resulting from the development of all possible compressibility effects at high g. Further, that shock stalling automatically tends to limit the g which can be applied in the trans-sonic range.

In air warfare of the future, victory will presumably go to the aircraft which can climb fastest and be manœuvred most handily at the highest possible speeds. This argues, among other things, the best possible thrust/weight ratio and consequent restriction of weight to a point where, while strong enough to withstand, say, 12g under normal conditions, it may be necessary (since the total loadings, changes in distribution of load and sudden changes in loading caused by bumps, buffeting, and local shock stalling, particularly at high g, may be very considerable in the trans-sonic range) to accept a lower safe limit of g at high Mach numbers, at least at low altitudes, and to some extent to sacrifice strength to performance.

3.2. The fitting of a Mach meter, while taking care of the situation in straight-line flight and possibly during very gentle manœuvres (*i.e.* when the g does not exceed say $1\frac{1}{2}$) may not then meet the case during manœuvres imposing considerable g since at the higher angle of attack the onset of

*In this connection, with an eye to the future, it is of interest that there is reason to expect a recovery of temperature in the stratosphere such that sea level temperatures, and so the same true speed of sound, are again reached at about 160,000 ft.

shock wave generation occurs at a somewhat lower Mach number.*

It may become necessary therefore to impose a sliding scale of maximum permissible g, decreasing from the 6g at normal Mach numbers down to say, 1½g (or whatever the safe maximum limit may be in any given case) at the limiting Mach number. This would be particularly necessary in order to provide for the case of a zoom started at or near the critical Mach number, in the course of which manœuvres are executed while height is being rapidly gained, and in which the Mach number of flight may be increasing, notwithstanding the fact that the indicated air speed may be falling.

3.3. If g limitations are to be imposed, since, as we have seen, the pilot is unable to sense small differences in applied g, it will be necessary to provide the pilot with an accelerometer or g indicator, and he will then have two instruments to watch concurrently! At the present time tests are being made with the object of developing a suitable type of accelerometer and of fitting such an instrument, should it prove necessary.

3.4. The necessity for watching two instruments concurrently will clearly impose upon the pilot an additional strain in circumstances, *e.g.* during high-speed combat manœuvres, such that his whole attention is likely to be focused on keeping the enemy in his sights and will probably prove unacceptable. Therefore, it will be necessary for a combined Mach number and g indicator to be developed. Such an instrument could conveniently have a single needle actuated by suitable linkage interconnecting a Mach number responsive element with a g responsive element, in such a manner that the needle would move over a red sector if, (a) in straight-line flight the Mach number reached the limit or, (b) in manœuvres, the g at the instantaneous Mach number became excessive (or vice versa).

There is one further complication introduced by diving conditions. If in a straight-line dive the Mach number of flight reached the limit, the application of any g in excess of the low value—we have assumed above 1½g—would immediately throw the needle into the red sector. This means that the pilot might find it virtually impossible to recover from the dive without exceeding the safe Mach-cum-g limit, especially should there be a tendency for g to build up during the pull-out, and with aircraft the terminal diving velocity of which is such that the "recovery height" (*i.e.* the height at which the terminal air speed falls below the critical Mach number allowing for the necessary g load) is too low to leave ample height for safe recovery.

3.5. To meet this it might be sufficient to add to the dial of our instrument an additional coloured sector—say yellow—the lower edge of which would correspond to the maximum practicable safe Mach number in a straight-line dive. This lower limit could then be fixed at a value such that the application of reasonable g—say 3 to 4g—would be possible without bringing the needle into the red sector.

3.6. As an alternative to an instrument comprising a needle moving over a dial, it would probably be better to have a red light arranged to come on at the danger threshold (or a series of lights arranged to come on in stages—one of which could correspond to the maximum straight-line diving limit—as the danger threshold is approached). Such lights could probably be embodied in the gunsight, or otherwise so positioned as to be clearly visible and well within the pilot's line of sight while he was concentrating on an enemy aircraft. The development of such an instrument should not prove an insuperable problem as both the essential actuating mechanisms are already available, although

*The reduction in critical Mach number per g depends upon the wing section and so on, but data available for a well-designed wing suggests that it will be between .005 and .01 M per g. Airframe distortions effects, however, may further increase the reduction in M per g.

possibly not yet in a sufficiently accurate form.

3.7. To ensure that in performing certain aerobatic manœuvres the indicated air speed does not drop to the stalling speed at the g being applied, it is necessary for such manœuvres to be started at not appreciably less than the speeds recommended.* These speeds are frequently quoted in the Pilot's Notes for the type and, since both the lift and indicated air speed are functions of density, it follows that such indicated air speeds apply to any height and cannot be related to fixed Mach numbers. The same applies to the normal stalling, approach and landing speeds, as well as to the maximum range and endurance cruising speeds usually quoted. For these reasons it will probably be necessary for a normal airspeed indicator to be retained as well as a Mach number—g (MG for short) indicator.

3.8. Should this be considered unacceptable for use at high speeds and in combat, there would appear to be no fundamental difficulty in adding to the MG indicator additional coloured sectors or lights arranged to indicate any required combinations of indicated air speed and g, independent of the actual Mach number of flight. These indications would be responsive to the accelerometer and air speed elements of the MG indicator only, the altitude responsive element of the Mach meter mechanism being inoperative for this purpose. The retention of a normal airspeed indicator (or combined I.A.S. and M.G. indicator) would, however, still be desirable for navigational purposes and for use during take-off, approach and landing, and possibly at high speed at low altitudes—see para. 2.3.

*In this connection it is of interest to note that on certain aircraft some manœuvres cannot be performed above certain heights, since the minimum safe speed for them is then above that corresponding to the critical Mach number. Further, certain manœuvres cannot be begun below an altitude such as will allow an ample margin of height for recovery. The higher the speed at which such manœuvres are performed, the greater the potential loss of height during, and in recovery from, the manœuvre.

PART II.

SOME POSSIBLE PROBLEMS OF THE MORE REMOTE FUTURE.

1. THE TRANS-SONIC REGIME.

1.1. In Part I we have considered the immediate problem presented by types of aircraft already developed or likely to reach the production stage within a year or so. Looking a little farther ahead, we have to consider the possible behaviour of aircraft in the trans-sonic and even supersonic speed ranges.*

1.2. In these conditions the chief advantage of a more or less conventional aircraft, as against a rocket or jet-propelled projectile type, would appear to be its superior power of manœuvrability and so ability to intercept and destroy the less manœuvrable rocket bomb or bomber.

1.3. So far there is no authentic record of any aircraft reaching a speed substantially in excess of its critical Mach number, with the exception of a Spitfire, which is authoritatively reputed to have reached a Mach number of .89 in a dive as early as 1943. It is of interest to note that the highest Mach number reached by the Meteor during its record-breaking runs was about .809. This was at low altitude and there is reason to believe it has reached higher Mach numbers at altitude.

1.4. Unfortunately, while supersonic wind tunnels have already been used successfully—notably in Germany—the difficulty of producing steady flow conditions in the trans-sonic range has made it impossible to obtain accurate data on the aerodynamic performance of aerofoils in these conditions. It is probable that between certain Mach number thresholds the lift coefficient may be

*An American aircraft project is stated to be stressed for 18g. Allowing a factor of, say, 1½ this will permit manœuvres imposing up to 12g and will necessitate the pilot wearing an anti-g suit. This aircraft is expected to exceed the speed of sound by a considerable margin; restriction of g at high speeds is contemplated and an accelerometer will be essential.

so far reduced as to make it impossible for the aircraft to maintain height. Should this be the case, it will only be possible to reach supersonic speeds with an aircraft having sufficient thrust to enable it to climb vertically, or nearly so, or by climbing the aircraft at subsonic speed and accelerating through the trans-sonic range in a dive; but, since the increase in true speed would have to allow for the decrease in Mach number with reduction in altitude, the rate of acceleration and/or loss of height involved would probably need to be very great.

1.5. For the reasons already discussed it may be assumed that the loss of lift of the control surfaces or, if these remain effective, the necessity of restricting g loads under the stress conditions resulting from the considerable increase and change in distribution of air loads on the structure, may so reduce the capacity to manœuvre in the trans-sonic range as to offset the advantages of this type of aircraft.

2. THE SUPERSONIC REGIME.

2.1. Should it prove practicable for speed to be increased, in a straight-line dive if necessary, to a Mach number well above 1.0 (to, say, 1.2 to 1.4) wind tunnel tests indicate that lift will again return while the drag coefficient in straight-line flight may be expected to fall rapidly, the distribution of load to be restored and the critical stresses reduced or stabilised, so again making the control surfaces effective and leaving a margin of strength to enable g loads to be safely applied once more. Thus, at speeds between the critical Mach numbers of, say, .85 and 1.2 it may be necessary to restrict aircraft to only straight-line flight, although at higher Mach numbers manœuvres may again be permissible.

2.2. Assuming that the Mach number responsive element of our MG indicator can be made to function satisfactorily through the trans-sonic range and above (there is considerable doubt as to the practicability of this at the present time), it should be possible to arrange for the instrument to take care of such requirements on the following lines. As the critical Mach number was approached in straight-line flight, the yellow light would show and if in the trans-sonic range an attempt were made to apply any appreciable g load, the red light would immediately come on. At Mach numbers above, say, 1.2, the yellow light would go out and it, and the red light, would then only come on during manœuvres imposing marginal or excessive g respectively.

2.3. The necessity for imposing any upper limit of speed in straight-line flight is problematical. In the drag conditions, even at the terminal velocity in a vertical dive at full thrust, the stresses in the structure may well not exceed those produced in the trans-sonic range at a lower total drag by the abnormal moments resulting from the changes in distribution of loading.

2.4. One interesting point is that at Mach numbers in excess of, say, 1.2, recovery from dives should be quite possible, provided that (a) the applied g did not exceed a safe figure for the conditions, (b) that the dive was not continued at a steady air speed, until the Mach number fell below, say, 1.2 or, (c) during recovery, air speed was maintained and the Mach number did not fall below this value.

2.5. If and when such aircraft—as distinct from rocket-propelled projectile types—become a technical practicability, there is little doubt that the provision of pressure cabins and suitable temperature control will overcome the altitude and temperature problems, at least up to 50,000 or 60,000 ft.; and that pressure anti-g suits will enable the pilot to stand up to any g loads which the aircraft could stand. With an anti-g suit, however, the pilot has even less sense of g than without; he would, therefore, be able to impose dangerous g without realising it and an accelerometer would be essential.

2.6. Whether the human brain will be capable of coping with the problems involved sufficiently rapidly in such conditions, would appear to be far more problematical. Intercepting and bringing an aircraft within combat range when the initial relative speed is of the order of, say, 2,000 m.p.h. would appear to involve rapid mental arithmetic of no mean order. Take the case of an interceptor pilot flying at 1,000 m.p.h. (true speed) spotting an enemy bomber (visually or by radar) when 10 miles away. He would first require to estimate the enemy's speed and course and then to decide on the best action to take to bring him within range at the earliest possible moment.

Let us assume as a simple example that he has means of estimating the enemy's speed (and that this is 1,000 m.p.h. true), and that he is approaching head-on at the same altitude. The chances of effecting a kill, using present types of guns, at a relative speed of 2,000 m.p.h. is clearly too remote for consideration, and the pilot would therefore require to turn so as to close with the enemy at a low relative speed. If he does so immediately at a rate imposing not more than a steady 4g, by the time he has turned through 180° (which will take about 40 seconds) he will be about 7 miles in a straight line from his starting point and travelling in the opposite direction. In the meantime the enemy will have travelled about 11 miles, and if he has not changed course the two aircraft will be about 7 miles apart, travelling on parallel courses. It should then be possible for the interceptor pilot to converge and, assuming he has a reasonable margin of excess speed over the bomber, to manœuvre within effective striking range in a matter of a further 60 seconds (15 miles).

2.7. Such almost ideal conditions are unlikely to be met with in practice. The bomber's initial speed and course may not be accurately known, his course and altitude may be changed, while the interceptor's g limitations may prevent his converging by the usual process of "keeping the enemy in his sights", *i.e.*, by aiming his aircraft straight at the enemy the whole time. Accordingly, he would have little alternative but to estimate a time and place of interception and then so to manœuvre as to bring his aircraft to the estimated rendezvous; in view of all possibilities of error, such a process is most unlikely to succeed.

2.8. This suggests that the only practicable solution may prove to be for a "pilot" at a fixed point on the ground to watch by radar the changing positions of the interceptor and bomber, and manœuvre the interceptor by remote control to bring it into engagement with the bomber. The need for a human pilot in the interceptor may then disappear entirely, unless he is retained merely to take over control when within striking range and to operate the armament.

2.9. Should this prove to be the ultimate solution, it will be necessary for our MG indicator to transmit its signals by radio to the "pilot" controlling the aircraft from the ground, or to provide other means of ensuring that the MG limitations are not exceeded.

ACKNOWLEDGMENT.

The author is indebted to Group Captain Clogstown and Messrs. Allwright and Charnley of the Ministry of Supply and the Royal Aircraft Establishment for their assistance in the preparation of the paper. The views expressed are the author's own and not necessarily those of the Ministry of Supply.

THE ROYAL AERONAUTICAL SOCIETY

THE USE OF RHEO-ELECTRICAL ANALOGIES IN CERTAIN AERODYNAMICAL PROBLEMS

by

L. MALAVARD

M. Malavard is Research Professor at the French National Centre of Scientific Research and is attached to the Technical Director of the National Office of Aeronautical Study and Research. M. Malavard is one of the founders of the system of rheo-electrical analogies and has applied it with considerable success to aerodynamics.

A MEETING of the Royal Aeronautical Society was held in the Library of the Society at 4 Hamilton Place, London, W.1, on Tuesday 25th September 1945, at which M. L. Malavard, of the Laboratoire des Analogies Electriques, Paris, lectured on "The Use of Rheo-electrical Analogies for the solution of Aerodynamical Problems." In the chair, Mr. E. F. Relf, C.B.E, F.R.S., F.R.Ae.S. (at that time Superintendent of the Aeronautics Division, National Physical Laboratory).

Lieut.-Col. J. Valensi (Liaison Officer for Aeronautical Research) acted as interpreter.

Mr. Relf: It was not often that members of the Society had the opportunity of hearing a lecture from a representative of one of the Allied countries, and that evening they were all helping forward the understanding between England and France.

The arrangements were made by Lieut.-Col. Valensi, who had been working in this country for some time and was anxious to extend scientific relations between British Scientists and their colleagues in France.

Lieut.-Col. Valensi (Associate Fellow): When he escaped from France in 1942 and reached Great Britain, he was welcomed as a friend; thanks to Mr. (now Sir Ben) Lockspeiser and Mr. Garner (of the Ministry of Aircraft Production), to Sir Charles Darwin (Director of the National Physical Laboratory), and to Mr. Relf (then Superintendent of the Aerodynamics Division, N.P.L.), he was given the opportunity to take his share in the common struggle by working at the N.P.L. At the Laboratory he had seen how important the work was which was done there and how carefully it was supervised by the various committees. Great Britain's achievements in aeronautics provided sufficient proof of the efficiency of the system.

Following the liberation of France, it was felt that contacts should be resumed between the two countries, and he was entrusted with the duties of Liaison Officer for Aeronautical Research. That was done with the agreement and support of the Ministry of Aircraft Production and the French Air Ministry and of the French Scientific Mission in Great Britain, with Professor Rapkine as leader. Exchanges had started between the two countries, and various visits by French scientists to British scientific establishments had been organised. Reciprocal arrangements had also been made for British scientists, and several parties of British Officers had been to France.

It was in connection with that liaison that he had the privilege to be present at the meeting of the Royal Aeronautical Society, to listen to M. Malavard's lecture about the work he had done on electrical analogy with his team, under the supervision of Prof. J. Peres. It was hoped that the liaison would continue, so that Great Britain and France would maintain close scientific contact.

On his own behalf, and in the name of all the French scientists who had come to this country, he expressed gratitude to all who had helped in the task—to Mr. Lock under whose supervision he had worked at the N.P.L.; to the Scientific Officers of the N.P.L.; to Mr. Perring, of the R.A.E., and all the Scientific Officers there, who had welcomed the French scientists; to Wing-Commander McGregor, at the M.A.P.; to Mr. Vessey and the members of the staff at the M.A.P.; and to the Royal Aeronautical Society, which had asked M. Malavard to lecture.

Mr. Relf: British scientists would subscribe to what Lt.-Col. Valensi had said about liaison with the French scientists, and would do all they could to foster such liaison.

He was credited with having started, many years ago, work of the kind with which M. Malavard would deal; in 1924 he had traced the streamlines of the inviscid flow of a fluid by tracing the analogous problem of the equipotential lines in an electric field. The idea had come to him as the result of an experiment he had made earlier still, in the Physics Laboratory at College. In that earlier experiment he had used a circular pot, fitted with conductors and electrodes, and had explored the equipotential lines. In 1924 it had occurred to him that such work could be extended in order to arrive at something much more precise.

Then in 1930 Professor Peres, M. Valensi and M. Malavard, at Marseilles, had taken up the work; had repeated his experiments and had extended them considerably. They had applied the electrical analogy method not only to tracing the streamlines, but to a variety of other problems. Later, Professor Peres and M. Malavard went to Paris, where they had continued the work with much more ambitious apparatus and had developed it still farther. At the Faculty of Sciences, in Paris, there had been established a Department of Electrical Analogy, and he imagined it was the only example in the world of a separate Section bearing that title. M. Malavard would now read his paper.

WHEN, in 1932, Professor Peres advised me to study the work of Relf and Taylor it did not occur to me that one day I would be able to express my gratitude to the originators of this method. It is a pleasant duty to acknowledge today to what extent I am indebted to these authors.

It is unnecessary to describe in detail the principles of the method of electrical analogies. These principles are explained perfectly in the work of Relf and Taylor.

By the method of electrical analogies an attempt is made to represent the solutions to certain problems of mathematical physics which depend on Laplace's equation or more generally on an equation of the type

$$\frac{\partial}{\partial x}\left(\lambda\frac{\partial\phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial\phi}{\partial y}\right)=0$$

by the distribution of electric currents in conductors.

In the work pursued at the laboratory of electrical analogies of the Sorbonne we have attempted to visualise this method in its entirety in order to base on it a real "experimental calculus."

Electrical analogies give graphical representations of the fields under investigation in the form of an orthogonal network of equipotential lines and lines of flux. It is also possible, by direct measurement of

electrical quantities, to obtain numerical values. Finally, it is possible to extend the range of applicability by the use of mathematical analysis which allows suitable transformations of the initial problem.

Here I shall only try to illustrate the principal aspect of the method by a number of examples. Applications concerning a great variety of questions in mathematical physics have been made in the laboratory of Professor Peres, but I shall restrict myself to aerodynamic problems.

EXPERIMENTS AND EQUIPMENT.

The classical experimental set-up for recording the equipotentials in the electric basin is shown in Figs. 1a and 1b. An electric dipper (p) made of platinum wire (diameter 0.1—0.2 mm.) plunges vertically into the electrolyte and is connected across a voltmeter (V) to the movable contact of a potentiometer (P). P is connected in parallel with the supplying electrodes. The voltmeter will register zero when the dipper and the movable contact of the potentiometer are at the same potential. By displacing the dipper in the field it is thus possible to define, point by point, the equipotential corresponding to the potential of the movable contact, which is read on the graduation of the potentiometer.

The points obtained in this way are transferred on to a sheet of drawing paper by a mechanical link, shown in Fig. 1b. It consists of a carriage in the form of a T, rolling on two guiding and one supporting roller moving along two parallel rails; a rod moves perpendicularly to the rails along the long arm of the carriage. The exploring plunger

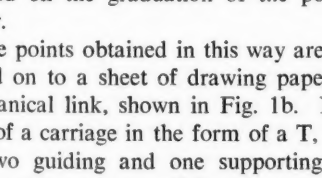


Fig 1a.
Layout for tracing equipotentials.

General Diagram of the Installation

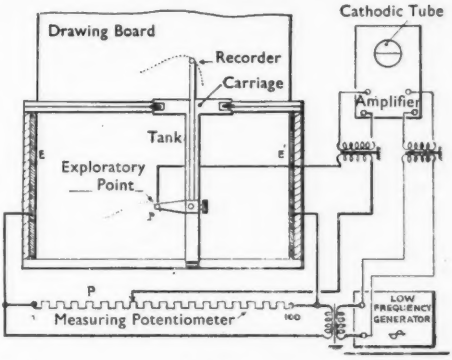


Fig. 1b.

and an electric inscriber, which perforates the paper in the points to be marked, are carried at the two ends of this rod respectively.

Alternating current is used to supply the electrolyte in order to avoid the phenomena of polarisation and electrolysis. For this purpose we use an L.F. oscillator, which produces a strictly harmonic current. In most of our experiments the frequency is of the order of a thousand cycles per second, but it is possible, without inconvenience, to reduce this to 500 or 400 cycles; in certain experiments we have even gone down to 50 cycles. The power used varies up to about 30 watts and the voltage up to about 100 volts according to the impedance of the circuit.

The diagram Fig. 1b shows that as voltmeter V we use a cathode ray oscillograph with amplifier. One of the two pairs of deflecting plates is connected to the potential difference to be made zero and the other to the electrodes of the supplying current; zero potential difference is obtained when the image is a straight line. This method of measurement is particularly suitable for rheo-electric experiments; it allows us to recognise errors due to polarisations or the capacities of parasitic induction, the straight line changing into an ellipse if the measured

tension shows a phase difference with respect to the general tension. Difficulties arising with this method can easily be avoided by special precautions; in some cases it is necessary to use alternating bridges (impedance bridge, Wagner bridge, and so on).

The liquid conductor contained in the electric basin is generally tap-water, a very convenient electrolyte, the conductivity of which is uniform even over large volumes.

We use a variety of electric basins; they are made of various insulating materials and are supported on adjustable mountings in

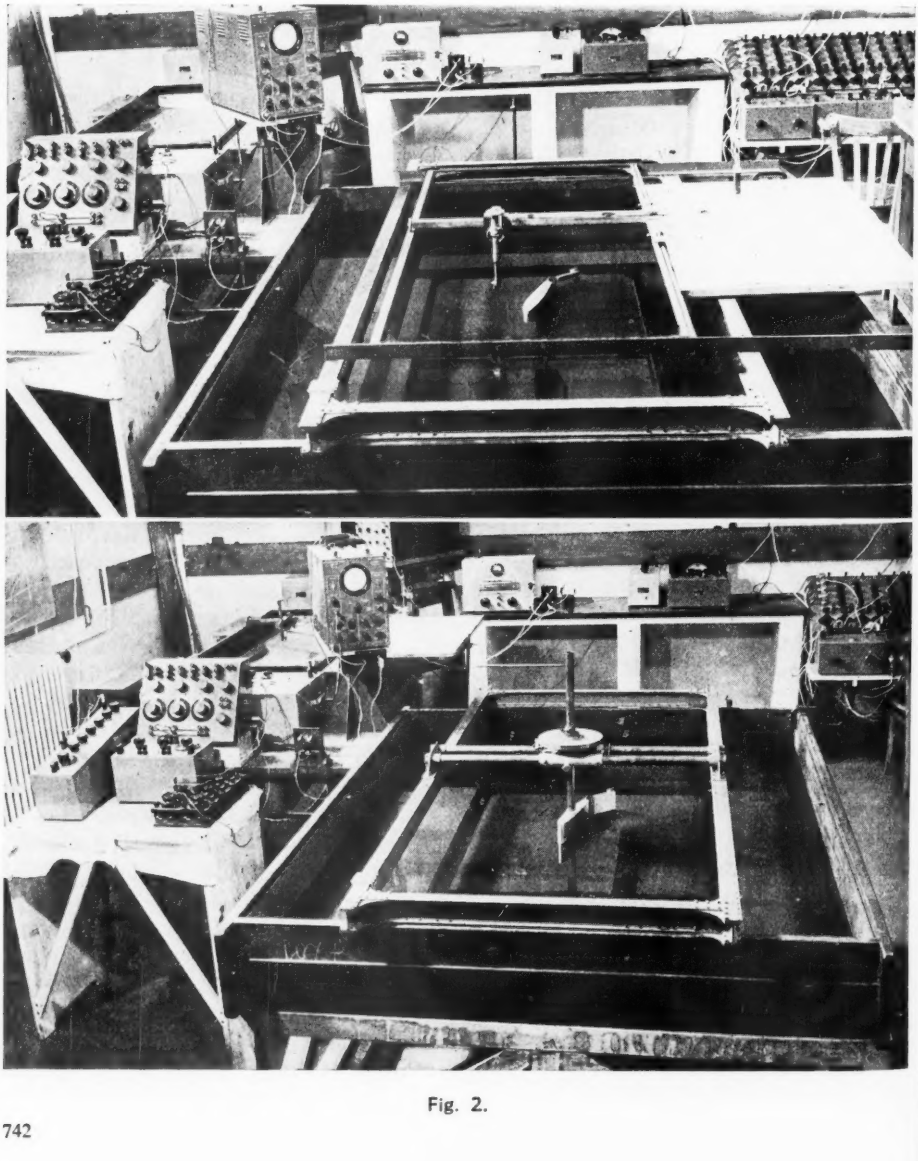


Fig. 2.

order to keep the bottom surface horizontal. Fig. 2 shows the largest of these two containers in use, made of slate and measuring 2 m. by 1.5 m. In such a container the equipotentials are plotted to an accuracy of 0.1 mm., measuring the potentials to an accuracy of 1 in 10,000.

This large container is generally used for the study of uniform flow around a wing profile. The equipotentials of the electric trough can be identified either with the hydrodynamic equipotentials (Direct Analogy A of Taylor) or with the lines of flow (Analogy B of Taylor) (2). I described, in 1934, a method of measuring directly the circulation round one or several profiles (3 and 4). In the second analogy the circulation corresponds to the total current flowing into the profile which is made of conducting material; in order to satisfy the Kutta-Joukowsky condition (finite velocity at the trailing edge) the model is equipped with a conducting dipper placed parallel to the trailing edge and very close to it on the bisector of the trailing edge angle. The oscillograph is connected in series between this dipper and the profile and the current flowing into the profile is adjusted to produce a zero reading; the value of the current now flowing into the profile gives the circulation. Fig. 3 shows two profiles equipped with their dippers; the lower model is one of a slotted flap: the adjustment of the circulation is produced simultaneously for the wing and the flap.

The characteristic values of a wing profile, namely the slope of the lift curve and the angle of zero incidence, can be obtained very simply by this method as the result of direct measurement. The angle of zero lift is found by determining that position of the model for which zero current flows into it and for which the trailing edge dipper is at the same potential.

The accuracy of these measurements is verified by comparison with the theoretical results of reference 4. The method constitutes a quick and practical way of obtaining empirical results for single sections and multi-planes.

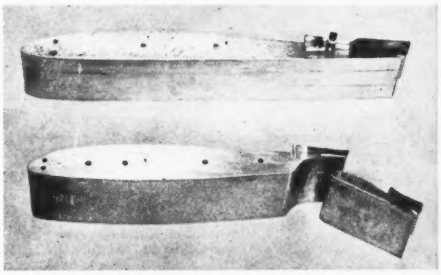


Fig. 3.

By means of two dippers at a small distance (1 to 2 mm.) from each other it is possible to measure the potential gradient and thus the velocity of the field at any point. The double dipper used for this purpose is shown in Fig. 4.

In order to measure the velocity on the contour of the profile it is possible to insulate electrically a small portion of this contour and to measure the corresponding element of circulation, in other words the current flowing into the small insulated portion. In this way the curves of Fig. 5, giving the pressure distribution on a flap for various

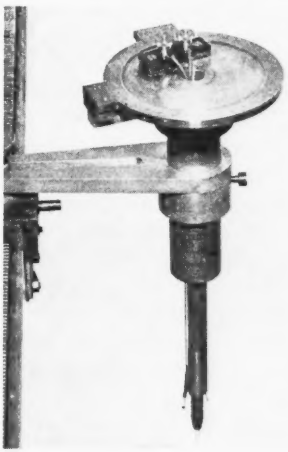


Fig. 4.

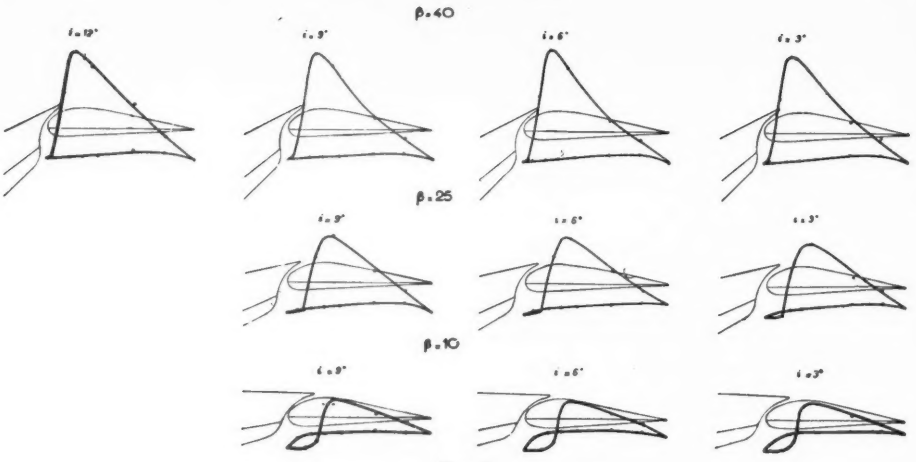


Fig. 5.
Distribution of pressure on the flap.

values of flap angle and incidence have been obtained; these results are fairly old but at the time we found them interesting because they showed that for a given flap angle this distribution appeared independent of the incidence.

These last experiments on pressure distributions are fairly delicate and for other applications we have investigated the possibility of obtaining experimentally the conformal representation of any profile on the exterior of a circle (5). The principle of the method is very simple. It is only necessary to consider a complex potential $\phi+i\psi$ defined in the region outside the profile and hence, by the conformal representation, outside the circle: if, by calculation or experiment in the electric basin, it is possible to plot the lines of ϕ and ψ on the one hand around the profile, and on the other around the circle, then the conformal representation is known because corresponding points correspond to the same values of ϕ and ψ in the two diagrams.

The simplest experimental realisation of this idea is to use the complex potential of a uniform flow. This potential can be immediately calculated in the case of the circle $\left(\phi+i\psi=Z+\frac{a^2}{Z}\right)$ and is easily determined around the profile by the electrical method. The two analogies (Taylor's A and B) are obtained by two successive experiments, one with an insulating, and the other with a conducting model (see Fig. 6), thus allowing us to plot ϕ and ψ. Simple formulæ now determine a point in the plane of the circle corresponding to each measured pair of ϕ, ψ. The fundamental quantities of the profile, namely, its axis, focus and the slope of the lift curve, are thus found with great accuracy. Incidentally, it may be noted that one single experiment with a conducting model suffices, if only the correspondence of the contours is required. The velocities around the profile obtained in this way have an accuracy of about 1 per cent. It is possible to visualise similar applications to a system of profiles or slotted flaps.

The application of this method yields a practical method of investigating the cascade of profiles which represents the blades of a rotor with axial flow (Fig. 7) (6). The cascade is defined by the profile thickness t, their spacing a and their incidence ω. It is only necessary to determine the ratio of velocities upstream and downstream V_1/V_2 and the corresponding deflection $\delta=\beta-\alpha$: from these all the characteristics of the cascade can be calculated.

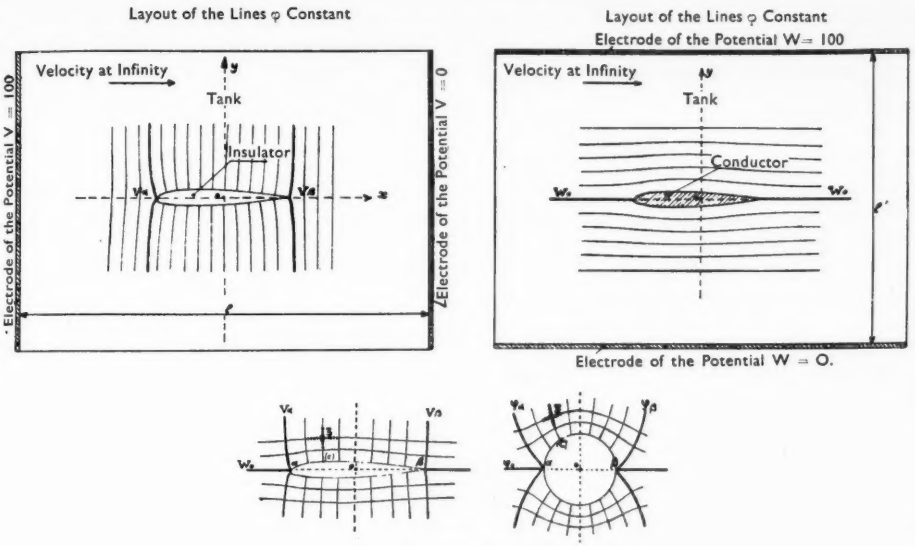

Layout of the Lines φ Constant
Velocity at Infinity
Tank
Insulator
Electrode of the Potential V = 100
Electrode of the Potential V = 0
Layout of the Lines φ Constant
Electrode of the Potential W = 100
Velocity at Infinity
Tank
Conductor
Electrode of the Potential W = 0.
Relation of the Two Fields

Fig. 6.

It is well known that the transformation $Z = e^{2\pi z/a}$ transforms the flow problem in the z plane into a problem relating to a single profile in the presence of a combined source and vortex at the origin in the Z plane.

If now the conformal representation of the exterior of the profile C on the exterior of a circle γ in the ζ plane is known then the problem is solved: it is sufficient to determine the flow around the circle γ due to the source-vortex now at the point M_0 of the ζ plane and to calculate the value of the circulation in such a way that the velocity at the point π which corresponds to the point P is zero.

It is in this last transformation (Z, ζ) that the rheo-electric method described above is used. In the usual case when only the overall characteristics of the cascade are required, it is sufficient to find the position of M_0 with respect to the circle, which is obtained very quickly by two experiments with the electric basin using the two analogies. It is merely necessary to determine in each case the value of the electric potential at the point of the basin which corresponds to the origin $Z = 0$: from these one determines the coordinates (ξ, η) of the point M_0. If, on the other hand, one wishes to obtain the full distribution of velocities, it is necessary to specify completely the one-one correspondence between C and γ.

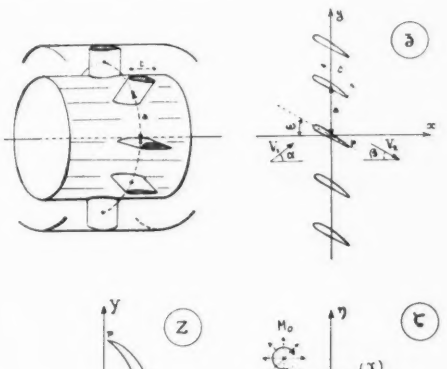


Fig. 7.

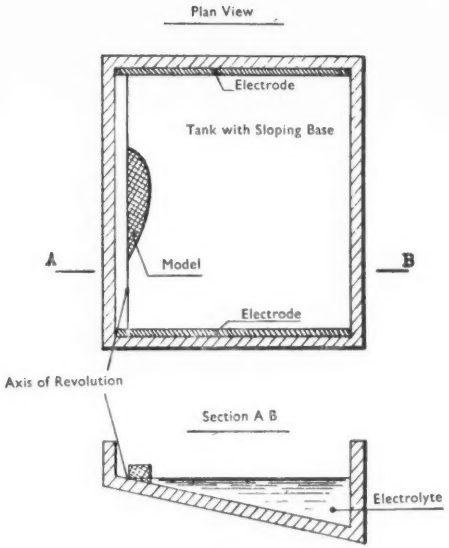


Fig. 8.
Apparatus for flow around bodies of revolution.

It is well known that the method of rheoelectric analogies can be easily generalised by using basins of slowly varying depth (Taylor, 2), and I shall return to this point presently with regard to a new application.

We are at present using a basin with an inclined bottom surface for the investigation of flow around bodies of revolution or in the interior of ducts and jets; the axis of symmetry of a body of revolution is represented by the edge of the liquid wedge formed in the basin. Fig. 8 shows the diagram and Fig. 9 gives an idea of our experimental set-up for this type of problem. Finally, Fig. 10 is an example of the equi-

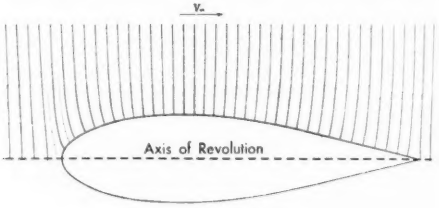


Fig. 10.

potentials for flow around a body of revolution.

THE "WING CALCULATOR."

One of the principal applications of the electrical analogy has been the "Wing Calculator," which enables one to solve the

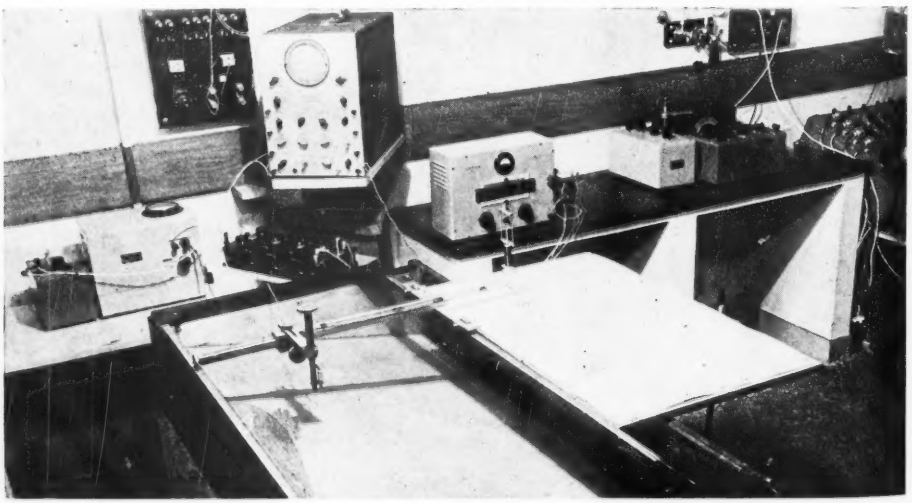


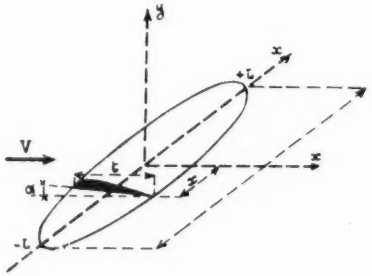
Fig. 9.

fundamental equation of the lifting wing, namely

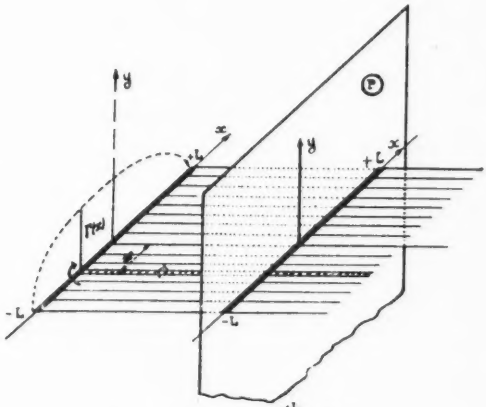
$$\Gamma(x)=ktV\left(\alpha-\frac{1}{4\pi V}\int_{-L}^{+L}\frac{d\Gamma(\xi)}{x-\xi}\right) \qquad (1)$$

(the notation is explained in Fig. 11) and thus to find the lift distribution for a given wing. References 3 and 4 contain the formulæ in general use.

The principle of the method is to obtain the field of the free vortices produced by the wing. It is known that at an infinite

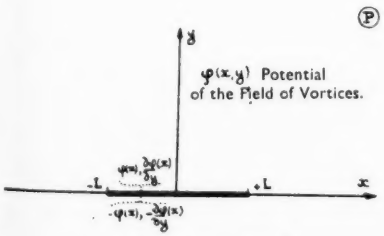


Data at Abscissa X $\begin{cases} t(x) \\ \alpha(x) \\ k(x)=\frac{1}{2}\left[\frac{dc_z}{d\alpha}\right]_{\lambda=\infty} \end{cases}$



$$\Gamma(x)=ktV\left(\alpha-\frac{1}{4\pi V}\int_{-L}^{+L}\frac{d\Gamma(\xi)}{x-\xi}\right) \qquad (1)$$

Flat Section of the Layer of Free Vortices



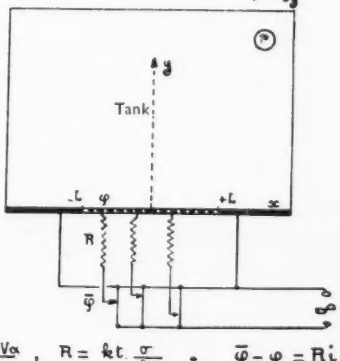
$$\varphi(x)=\frac{1}{2}\Gamma(x) \quad \text{et} \quad w(x)=-\frac{1}{2}\frac{\partial\varphi}{\partial y}$$

$$?\,\varphi(x)=ktV\left(\alpha+\frac{1}{2V}\frac{\partial\varphi}{\partial y}\right) \qquad (2)$$

Diagram of the Electrical Apparatus
"Calculator for Wings."

φ Electric Potential.

i. Intensity of Electric Current. $=-\frac{k\,dx}{\sigma}\frac{\partial\varphi}{\partial y}$



$$\bar{\varphi}=\frac{ktV\alpha}{2}\,,\quad R=kt\frac{\sigma}{4k\,dx}\,,\quad \bar{\varphi}-\varphi=Ri \qquad (3)$$

Fig. 11.
Wings of the Finite Span.

distance downstream this field is plane and can be investigated by considering a section at right angles to the vortex sheet (Plane P of Fig. 11). In this plane it is easy to find the potential $\phi\ (x, y)$ of the free vortices and by considering the line $(-L+L)$ to relate the circulation $\Gamma(x)$ and the induced velocity $w(x)$ to the potential $\phi(x)$ and its normal derivative $\frac{\partial\phi}{\partial y}$. Equation (1) then reduces to

$$2\phi(x)=ktV\left(\alpha+\frac{1}{2V}\frac{\partial\phi}{\partial y}\right) \quad . \qquad (2)$$

An electric image of this field is obtained by representing the upper half plane by an electric basin, one side of which represents the axis Ox and is supplied with current by electrodes covering a segment corresponding to the wing span $(-L+L)$. In order to satisfy equation (2) on each electrode, the potential $\phi(x)$ being identified with the

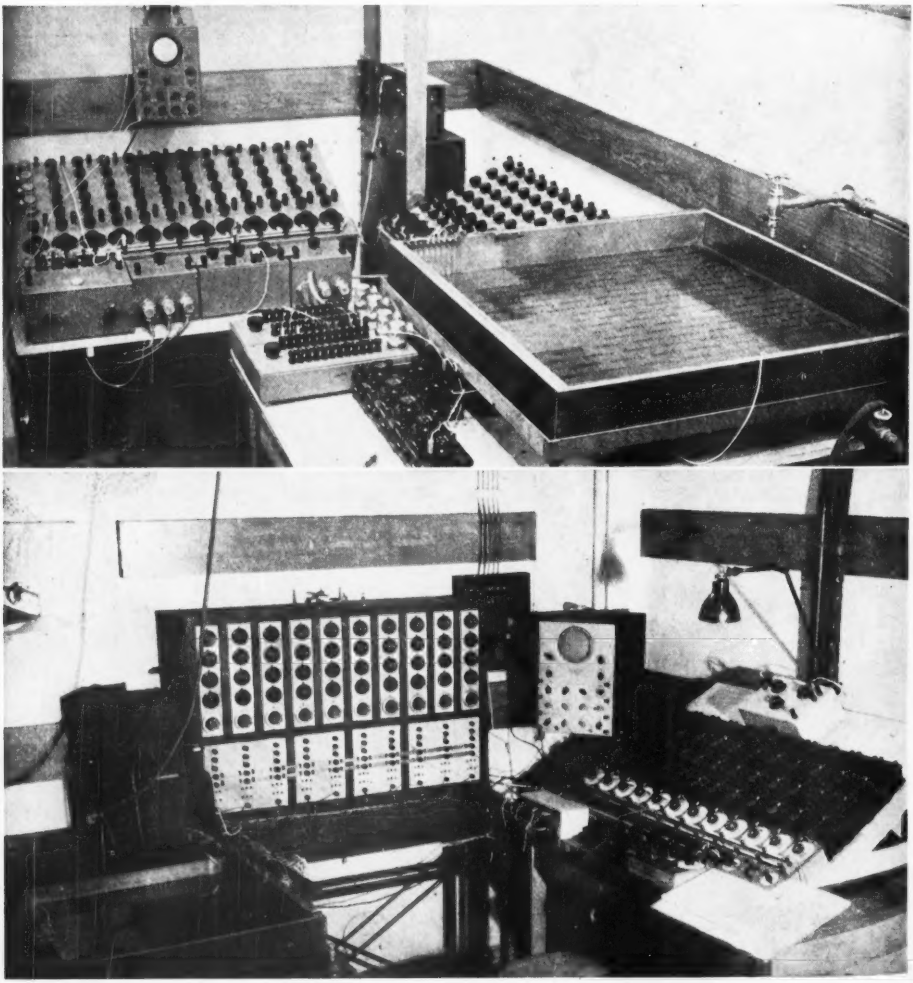


Fig. 12.

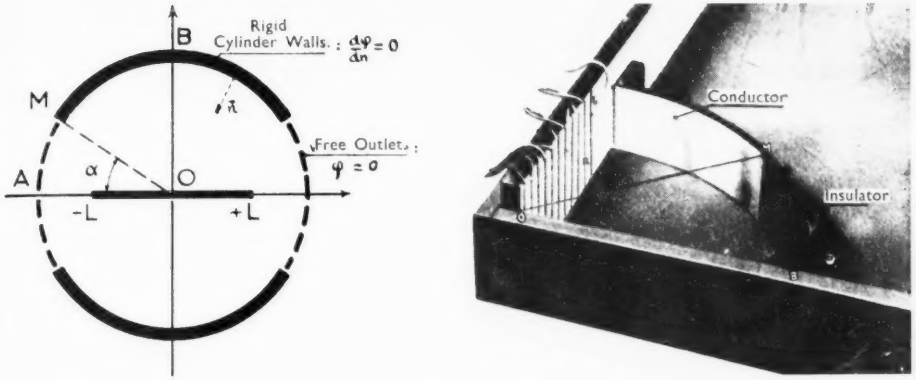


Fig. 13.

electric potential and its normal derivative with the current, the current passes through a resistance R to obtain given values $\bar{\phi}$ of the potential. It is easy to verify that R and $\bar{\phi}$ are determined by the given quantities $t(x)$, $z(x)$ and $k(x)$.

I have described the full scope of this method in reference 4. It constitutes a notable extension of the method of analogy: while the application to problems of the Dirichlet or Neumann type (first and second boundary value problems of potential theory) is immediate, the wing calculator enables problems of the Fourier type (third boundary value problem) to be solved. In this case the boundary condition can be written

$$\phi - H\frac{\partial \phi}{\partial n} = F \quad . \quad . \quad (3)$$

where H and F are given on the boundary or part of the boundary of the field.

Equation (3) reduces to Ohm's law

$$\frac{F-\phi}{H} = i$$

in such a way that one is led to a problem of Dirichlet type in the field extended by a heterogeneous conductor which only allows the current to pass in a given direction: we can imagine such a conductor formed by the juxtaposition of an infinity of linear-conductors having resistances chosen according to the values of H. This is the significance of the resistances R through which current is supplied to the electrodes of the wing calculator.

A variety of forms of the apparatus have been used since 1933 and Fig. 12 shows two of these. One main application made was the compilation of a table concerning the lift-distribution of 1,200 different trapezoidal wings.

The apparatus can also be used in the solution of other questions relevant to the linear-theory of the wing of finite span. For instance, to find wind tunnel corrections it is only necessary to limit the electrolyte in the basin by boundaries corresponding to the cross-section of the duct, the boundary being a conductor if we are dealing with an open tunnel and an insulator for a closed one; the case of a mixed partly open and partly closed tunnel, to give a zero correction is thus easily determined. Fig. 13 shows an example of this experiment. Another method is based on a conformal transformation of the tunnel section, which is then used in the electric basin (8).

It is possible to visualise the use of this method to treat generalised forms of wing theory leading to equations more complicated than that of Prandtl (equation 1), which can often be solved by iterative solutions of equations of that type. The ease of obtaining solutions by the electrical method

makes it particularly suitable for this purpose. Certain problems concerning the maximum lift of a wing have been approached by this method (9).

The method can also be applied to more complicated systems as multi-planes, wings with flaps, and so on. The cross-sections of the vortex sheet are then represented, as shown in Fig. 14, by insulating plates covered with electrodes on each side. These plates are placed in the electric basin and the attached electrodes supplied with current in a way similar to that described for the ordinary wing calculator.

More recently M. Siestrunck has succeeded in applying the principles of the wing-calculator to evolve an "airscrew calculator." The vortex theory of the airscrew shows that the potential ϕ of the induced velocities due to the vortices, which is defined in a three-dimensional field, depends only on two variables, namely, the radius vector ξ and $\zeta = \theta - \frac{z}{h}$ (see Fig. 15), where h is the pitch of the spiral of vortices. In this system of co-ordinates the potential satisfies a differential equation of the type mentioned in the introduction to this lecture, and thus an electrical representation is possible in a basin of variable depth, the determination of the depth itself being quite easy, as shown in Reference 10. Along the segment $(0, +1)$ which represents the blade, the potential ϕ satisfies an equation similar to that mentioned in connection with the wing calculator and accordingly the same set-up of electrodes and supplying resistances can be used. The influence of the number of blades is represented by the presence of a single straight electrode at the potential zero and making an angle π/p with the segment which represents the blade (see Fig. 16).

The wing calculator is a very convenient instrument, but it requires some personal experience and some preparation every time before it is used (at any rate it is necessary to fill the basin). It was, therefore, a very natural idea, which we succeeded in putting

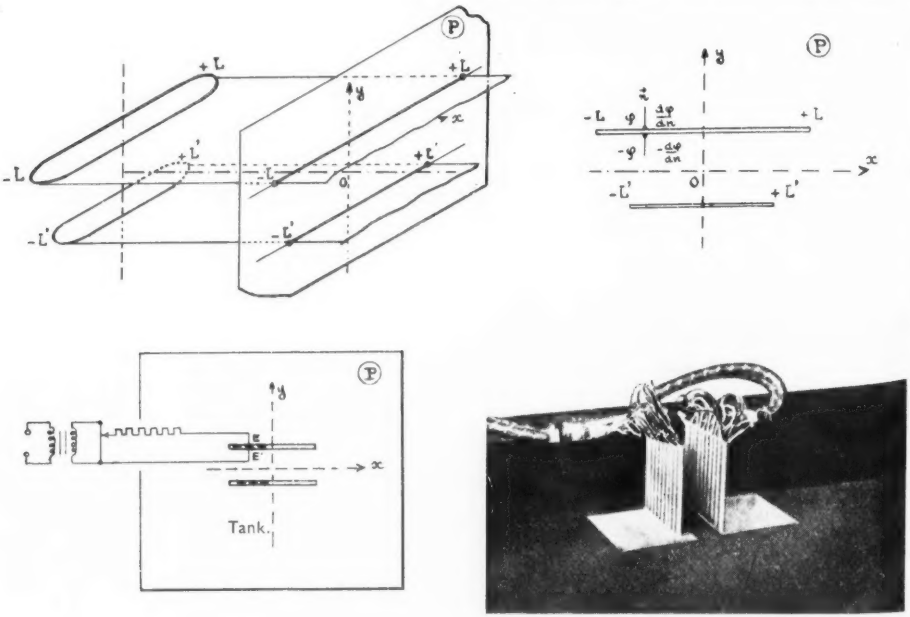


Fig. 14.

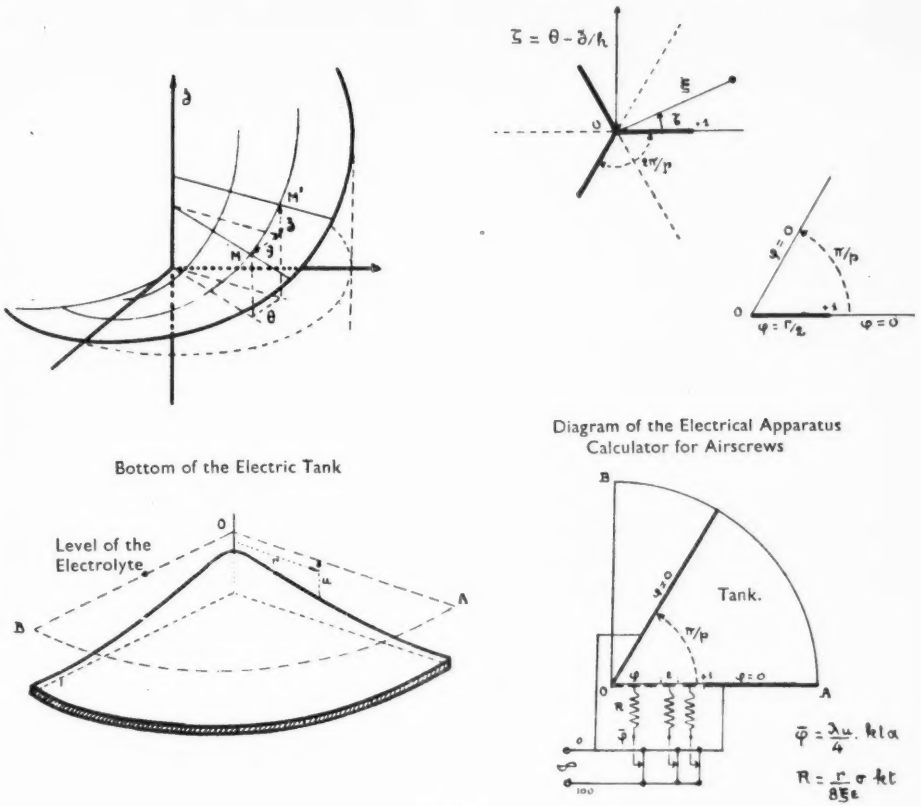


Fig. 15.
Theory of the Airscrew.

into practice in 1944 (13), to replace the electric basin by a network of linear-conductors which, once set up, is always ready to be used. The principle of this method is based on the following: The field in the basin can be considered as the super-position of elementary fields obtained by supplying current to each electrode individually. In this case the resistance between any two electrodes is quite definite and hence the basin can be replaced by a network of resistances connecting the electrodes in pairs (see Fig. 17 corresponding to 4 electrodes)*

The remainder of the experimental set-up remains unchanged.

The above constitutes a method for the electrical solution of any problem in which the given data concern only the boundaries of the field, provided only that one can determine either by experiment or by theory the resistances between each pair of electrodes (Reference 13 and 14).

The apparatus at present is used for various routine experiments and has proved to be very convenient. It does not replace the original wing calculator entirely, because it does not allow the finding of the field of free vortices, a knowledge of which is necessary for certain applications. On the other

* This photograph shows the network of resistances and, next to it, the board carrying the electrodes of the wing calculator.

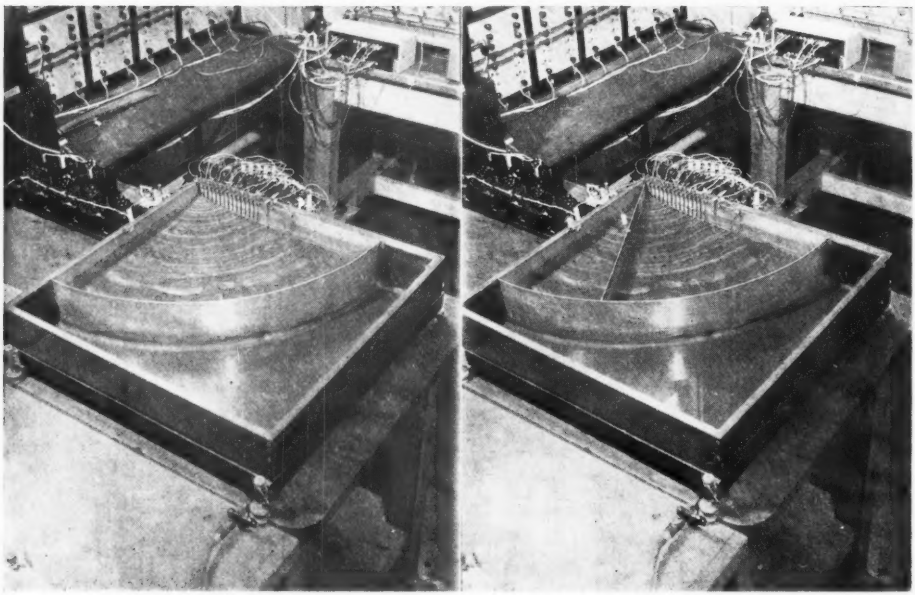
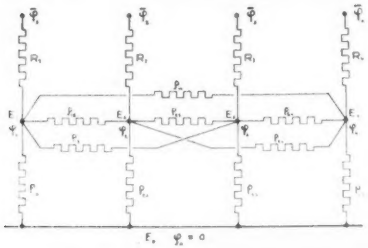


Fig. 16.

hand it has the advantage of working equally well with direct or alternating current.

This apparatus, consisting of a network



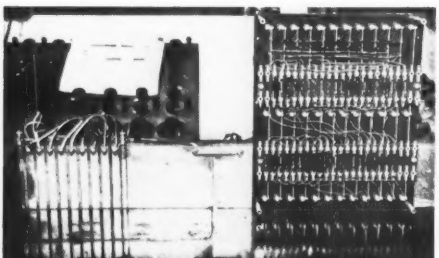


Fig. 17.

of regulated resistances, gives the immediate solution of a linear system of equations of the type

$$\alpha_p \phi_p = \Sigma' \alpha_{pq} \phi_q + \beta_p$$

where the ϕs are the unknowns, where all the α_{pq} are positive and satisfy the inequality $\alpha_p < \Sigma' \alpha_{pq}$. From this point of view the wing calculation, due to Multhopp, generally used in Germany, corresponds closely to the electrical method. After a simple transformation the equations used by Multhopp reduce to the above type. By choosing suitable resistances their solution is direct.

I hope I have shown that the interest is not confined to the use of one special method of analogy or more generally to any experimental method of calculation, but even in the restricted field of electrical analogies very different points of view are possible and can be applied: an example of this is given by the calculator without electrolyte, which acts as an accurate calculating machine, the use of which is by no means limited to the problems in which it originated.

I should like to thank all those who have honoured me by their presence at this lecture, and particularly Mr. H. M. Garner, D/DGSR of M.A.P., and Mr. E. F. Relf, who took the chair.

Finally, I would like to express my thanks to the Director of Scientific Research at the Ministry of Aircraft Production and to the Royal Aeronautical Society, which has organised this meeting.

REFERENCES

1. E. F. Relf.
An electrical method of tracing streamlines for the two-dimensional motion of a perfect fluid (Aeron. Res. Com. and Mem. et Philos. Magazine, 1924, t, 48, p. 535-539).

2. G. I. Taylor and C. F. Sharman.
Problems of flow in compressible fluids (Proc. Roy. Soc. of London, série A, t, 121, 1928).

3. L. Malavard.
Application des Analogies Electriques à la solution de quelques problèmes de l'Hydrodynamique (Publications Scientifiques et Techniques du Ministère de l'Air, fasc. No. 57, 1934).

4. L. Malavard.
Etude de quelques problèmes relevant de la théorie des ailes. Application à leur solution de la méthode rhéoélectrique (Publ. Scient. et Techn. du Ministère de l'Air, fasc. No. 153, 1939).

5. L. Malavard.
Sur la solution rhéoélectrique de questions de représentation conforme et application à la théorie des profils d'ailes (Comptes-rendus de l'Académie des Sciences, Jan. 1944).

6. L. Malavard et R. Siestrunck.
Sur une méthode d'étude des grilles indéfinies de profils quelconques (Communication au Congrès National de l'Aviation Française, April 1945).

7. J. Peres et L. Malavard.
Tables numériques pour le calcul de la répartition des charges aérodynamiques sur l'envergue d'une aile (Rapport technique No. 9 du Groupement Français pour les Recherches Aéronautiques, 1943).

8. J. Peres et L. Malavard.
Sur la détermination des corrections de soufflerie (Comptes-Rendus de l'Académie des Sciences, Sept. 1945).

9. J. Peres, L. Malavard et L. Romani.
Problèmes non linéaires de la théorie de l'aile. Application à la détermination du maximum de portance (Actuellement à l'impression: Rapport Techn. du Groupement Français pour les Recherches Aéronautiques).

10. R. Siestrunck.
Sur un mode de solution rhéoélectrique des problèmes de l'hélice propulsive (Comptes-Rendus de l'Académie des Sciences, Sept. 1944).

11. R. Siestrunck.
Sur les corrections de parois dans les essais d'hélices (Comptes-Rendus de l'Académie des Sciences, Sept. 1945).

12. R. Siestrunck.
Sur le calcul des helices ventilateurs (Comptes-Rendus de l'Académie des Sciences, Sept. 1945).

13. L. Malavard.
Application aérodynamique du calcul expérimental analogique (Commun. au Congrès National de l'Aviation Française, April 1945).

14. L. Malavard.
Calculateur d'ailes et réseau de résistances linéaire pouvant remplacer, dans certaines questions, le bassin électrique (Comptes-Rendus de l'Académie des Sciences, July 1945).

[Translated from the French by Mr. V. K. M. G. Gugenheim, formerly on the staff of the Royal Aeronautical Society.]

DISCUSSION

The Chairman: It was of great interest to note the way in which the electrical analogy methods had been developed, even to dealing with the potential flow problem around the screw, which was a complicated matter. It behoved scientists in Great Britain to get hold of the report of the French work and to take note of what had been done, for no doubt there were applications of that kind of method which they should be developing now. One of the aircraft firms had an electrolytic tank for dealing with revolving bodies, and one had been made also at the N.P.L.

Mr. H. M. Garner (M.A.P., *Fellow*): So far as he knew, the only company in this country which had been using the method as a practical means of obtaining information, was the Bristol Aeroplane Company; they had had a tank in operation for a considerable time, and had obtained some useful information in connection with the flow around nacelles.

They had a great deal to learn in this

country, and scientists from the R.A.E. and the N.P.L. were visiting France to see what was being done and to discuss the work with M. Malavard in order to obtain as much information as possible. He hoped that visits of British scientists to France and French scientists to Great Britain would become much more frequent than they had been in the past.

He believed that the first person to put the electrical analogy method on a really proper footing was G. I. Taylor, who had written two interesting reports, one in the Proceedings of the Royal Society, and one in the R. and M. series. Although those reports had dealt mainly with the application of the method to compressible flow, it was applicable to other problems.

He wondered to what extent the method could be applied to incompressible flow? The problems of flow around bodies in slightly subsonic and slightly supersonic regions were rather difficult to investigate in any way.

M. Malavard: The Taylor method for the representation of compressible flow involved variable depth; to achieve the variable depth one must proceed step by step, and change the depth of the tank.

Another method was the plan of the velocities, which did not involve variation of depth. At first sight that seemed simpler, but, unfortunately, the bottom surface became extremely complicated. There was some hope that those problems would be solved in the future.

Mr. R. A. Fairthorne (R.A.E.): For a century or so engineers had been trying to make mechanical analogies for electrical phenomena, and electricians had been trying to make electrical analogies for mechanical phenomena—it varied with the fashion—but only in a few cases had the work materialised into actual apparatus. The method of representing an elastic beam by a most complex collection of six-wire transmission lines was becoming tremendously popular on the other side of the Atlantic. He had great respect for anyone who had managed to make the analogies work in practice as electrical measuring devices. But what M. Malavard had omitted to point out was that a high degree of experimental skill was required, both in the design and the use of the apparatus, to make even the simplest analogy work. It was easier for people who had concentrated on the numerical side of things. He congratulated M. Malavard and his colleagues on the extreme skill which they had developed.

The tendency during the past ten years in electrical analogy methods had been to make networks, and he felt that that tendency had gone much too far. The network apparatus which had been developed and used in the United States since 1934 had reached an incredible degree of complexity, so much so that he would guarantee that the ordinary calculating machine would require less time to calculate numerically than would be required to set up the network apparatus.

In M. Malavard's type of analogy the network was of infinitesimally small mesh, which they could not hope to handle by numerical methods or by the network apparatus. M. Malavard had to make models; if he wanted rough results, to keep a few points in his plot of the flow, his accuracy was no worse than if he wanted a very fine plot, because it worked out its own equations. On the other hand, with the network apparatus or numerical methods a rough run over the field had to be done, which might be extremely misleading, because the accuracy depended on the mesh. Therefore, he considered the type of apparatus described by M. Malavard, when used by really skilled people, was of enormous assistance in conjunction with the numerical methods of refinement; clearly there was a limit to the accuracy which any physical instrument would give, whereas in theory with a numerical apparatus there was no limit, and a cheap calculating machine would give results correct to one part in 10 million. Although electrical analogy and

physical methods were usually opposed, there was no doubt that in conjunction they were most powerful.

It was worth remembering what lay at the root of all that work, the solution of simultaneous equations, not necessarily linear. In the case of M. Malavard's work there was an infinite number of variables, and in his own case there was a finite number of variables. In the usual numerical method the time required was roughly proportional to N^3, where N was the number of variables. Usually, if they had a flying start from physical analogy apparatus the time could be reduced as being proportional to N^2. What he did not know was the factor of proportionality. But there was an enormous difference, because if they were dealing with 100 or 1,000 unknowns, as they must do in the next few years, the difference between $1{,}000^3$ and $1{,}000^2$ was considerable in terms of time and money.

Dr. S. H. Preston (N.P.L., *Assoc. Fellow*): What degree of accuracy could be achieved by M. Malavard's method in measuring the speed of the flow over the surface of a wing? The investigators at the N.P.L. were interested in developing the accuracy of the method in order to save the use of the computing machines.

He was also interested in the application of the analogy method to flutter work. The work discussed in the lecture had been concerned with steady motion; he wondered whether the method could be applied to unsteady motion?

M. Malavard: By the direct method, using the two needles, an accuracy could be achieved to within 2 or 3 per cent., or to within 4 per cent. if the experimenter were not expert. By the indirect method an accuracy to within 1 per cent. could be achieved.

He could not solve the flutter problem by the analogy method.

Mr. R. G. Fowler (N.P.L.): He believed that the application of Taylor's method to compressible fluids broke down when the speed of sound was reached; could the method be applied to the supersonic case? He suggested that the shock waves might be represented by special razor edges, and then they might proceed in the same way as for the compressible fluid below the speed of sound.

M. Malavard: The tank could be used only when solving problems connected with the Laplace equation.

Mr. Vessey (M.A.P.): He had seen M. Malavard's apparatus in France and had been partly instrumental in getting him to visit this country. The practical details of the apparatus were most impressive. He hoped that those with whom M. Malavard discussed the problems, and the representatives of the R.A.E., who were going to France to follow up the work, would take careful note of those practical details, for he believed that quite a lot of the success of the method depended upon them.

Although M. Malavard had confined his lecture strictly to aerodynamics, there were other applications of his methods, for example, in regard to heat flow, which was of interest to the engine people. In fact, there was a wide application of his methods in almost exactly the same form as used for the solution of aerodynamical problems.

M. Malavard: Some of the further applications of the method had been appreciated. For example, the distribution of the stresses inside a body under torsional flexure could be determined, and the electrical method could be used as an addition to photo-elastometry, to assist in the analysis of the stresses. One of the difficulties that had to be overcome was the representation of the boundary conditions. But such problems could be solved.

Mr. J. Hadji-Argyris (Royal Aeronautical Society): Could the methods discussed in the lecture be applied to elastic problems represented by bi-harmonic equations?

M. Malavard: He had been interested in that particular field, but had experienced

great difficulties because of the boundary conditions.

The Chairman: He proposed a vote of thanks to M. Malavard for his most instructive lecture, and for presenting a subject which they had not explored nearly so far in Great Britain as in France.

Lt.-Col. Valensi: Professor Peres had wished particularly to attend the meeting, but his duties had prevented him from reaching England in time. He had done much of the mathematical part of the work discussed in the lecture and had inspired many of the settings. M. Malavard was to be congratulated for the manner in which he had achieved those settings.

THE ROYAL AERONAUTICAL SOCIETY

Paper submitted May 1947.

THE GENERAL THEORY OF CYLINDRICAL AND CONICAL TUBES UNDER TORSION AND BENDING LOADS

Single and Many Cell Tubes of Arbitrary Cross-Section with Rigid Diaphragms

PART V

by

J. HADJI-ARGYRIS, D.E., A.R.Ae.S., and P. C. DUNNE, B.Sc., A.F.R.Ae.S.

Parts I to IV of this paper were published in the February 1947 *Journal*

5. GENERAL ANALYSIS FOR ARBITRARY LOADING.

SUMMARY.

Part 5 continues the theory given in Parts 1-4 in the February 1947 issue of the Journal (pp. 199-269). The present paper deals with the stresses and deformations of conical and cylindrical tubes under arbitrary loading, thus completing the analysis given in Part 4 which dealt only with pure torque.

A remarkable unification of the theory of bending and torsion is achieved. It is shown that the axial constraint stresses, *i.e.* the corrections to the engineers' stresses, may in general be calculated as if caused by torque about axes different in each mode. The analysis proves that it is in no circumstances correct to calculate the axial constraint stresses from the torque about the flexural axis.

Functions giving the cross-wise distribution of stress are fully worked out for the n-boom tube of arbitrary cross-section and for the singly symmetrical trapezoidal tube with continuous direct stress-carrying covers. The analysis is a considerable extension of the results given for the four-boom tube in Part 4.

CONTENTS.

5. General analysis for arbitrary loading.
Summary.
5.1. Introduction.
5.2. Additional notation.
5.3. Additional references.
5.4. General theory for an arbitrary shear distribution.
5.4.1. The statically equivalent stress system for arbitrary loading.
5.4.2. The generalised engineers' theory stress systems $g_X h_X$, $g_Y h_Y$.
5.4.3. Determination of the h_X and h_Y functions for single and multi-cell tubes.
5.4.4. The differential equation in the g_i function for arbitrary load distribution.

5.4.5. The conical tube with $t^* = \rho^\alpha$ and $t_s^* = \rho^\beta$; the particular case when $\alpha = \beta$.
5.4.6. The conical tube with $t^* = t_s^* = \rho^\alpha$ under a concentrated load at $r = r_1$.
5.4.7. The built-in conical tube with $t^* = t_s^* = \rho^\alpha$ under a distributed load $\omega_x = \omega_{x0}\rho^{\gamma-2}$.
5.4.8. The cylindrical tube with linearly varying thicknesses.
5.4.9. The cylindrical tube of constant thickness longitudinally with a concentrated load at $z' = z_1$.
5.4.10. The cylindrical tube of constant thickness under uniformly distributed load ω_x between the root and $z' = z_1$.
5.4.11. The cylindrical tube of constant thickness longitudinally under arbitrary shear loading.
5.4.12. The particular case of eigenloads giving bending without twist.
5.4.13. Alternative procedures for determining ξ_i, η_i.
5.4.14. The deflections and twist of the tube under arbitrary loading.
5.4.15. Formulæ for stresses referred to any system of axes X, Y.
5.4.16. The warping function W_1^* for arbitrary loading.

5.5. Stresses and warping caused by an arbitrary self-equilibrating end load system.
5.5.1. The unit eigenloads or end load systems.
5.5.2. Development of an arbitrary self-equilibrating end load system in terms of unit end load systems.

5.6. The four-boom tube under arbitrary loading.
5.6.1. Stresses and deflections of four-boom tube under arbitrary loading.
5.6.2. Formulæ for ξ and η.

5.7. The n-boom tube in general.
5.7.1. How to form the secular equation for the n-boom single-cell tube.
5.7.2. The proof of the secular equation (397) for an n-boom tube.
5.7.3. The h-functions and the co-ordinates ξ_i, η_i for a single-cell n-boom tube.
5.7.4. Reduction of a multi-cell n-boom tube to an equivalent single-cell tube.
5.7.5. The multi-boom tube with single symmetry.
5.7.6. The multi-boom tube with double symmetry.

5.8. The singly symmetrical trapezoidal tube with direct stress carrying top and bottom panels.
5.8.1. The secular equation.
5.8.2. The h-functions and the co-ordinate η.
5.8.3. The h-functions and the value of a_s for flexure in the YZ plane.
5.8.4. The singly symmetrical trapezoidal tube with walls effective only in shear.
5.8.5. The doubly symmetrical rectangular tube with direct stress carrying top and bottom panels.

5.1. Introduction.

As stated in the introduction 1.1.2, p. 204 (February 1947 Journal) the purpose of the present part is to give the theory of cylindrical and conical tubes under arbitrary shear distribution. The analysis presented has achieved a unification of the theories of bending and torsion in a much more complete sense than the authors had at first envisaged. In fact, the axial constraint stresses caused by arbitrary loading may be calculated in general as corrections caused by torque. The only difference, between the general case and the pure torque case, is that the torque in each eigenload system is associated with the axis

of twist OS_i of the corresponding self-equilibrating end load system. The reader will remember that the axis of twist was the straight line joining the apex of the cone with the point ξ_i, η_i at the root cross-section.

The importance of these results is clearly brought out by considering the following typical formulæ for the stresses due to arbitrary loading,

$$\left.\begin{aligned}\sigma &= \sigma_E + \frac{1}{2A}\sum_i T_{Bio}\, G_{\sigma i}\, H_{\sigma i} \\ \tau &= \tau_B + \tau_E + \frac{1}{2A}\sum_i T_{Bio}\, G_{\tau i}\, H_{\tau i}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (314)$$

which should be compared with equation (1) *of* 1.1.2. The terms σ_E and τ_E are the direct and shear stresses given by the ordinary engineers' theory of bending as generalised for conical tubes, and τ_B is the shear stress as calculated from the Bredt-Batho theory of torsion. These stresses are a suitable system statically equivalent to the applied load. The coefficient T_{Bio} is the value of the torque due to the shear stress distribution at a fixed section (preferably the root) about the point S_i. Note that in the absence of shear $T_{Bio} = T_o$. The H-functions indicating the variation of the stresses over the cross-section are exactly as given in equation (138) of Part 4. In many cases the G-functions, which depend only on the loading and lengthwise variation of the cross-section, are the same as those developed in Part 4. *For example, the G-functions for a concentrated shear load through any point of a cross-section are identical with those for a concentrated torque at the same section.*

It is noteworthy that this unification of the theory of bending and torsion is only brought out through the assumption of an arbitrary cross-section. In fact, the effect of symmetry is to break up the eigenloads into separate groups with no apparent connection. Thus for cross-sections with double symmetry, one has to consider entirely separately the axial constraint stresses due to shear in the two planes of symmetry and torque about the axis of symmetry. Now the latter cases appear as limiting forms of the general formulæ (314), but an analysis starting from the assumption of double symmetry does not suggest the essential unity of all cases and the proper procedure for arbitrary cross-sections. It is clear from the present theory that the practice of calculating axial constraint stresses on the basis of torque about the flexural axis is completely wrong, since the positions of the axes OS_i may differ widely for each mode and have no relation to the flexural axis.

The detailed analysis leading to the above conclusions is given in 5.4.1 to 5.4.13. The G-functions for many important loading cases have been fully worked out for a built-in tube with tip free.

The remainder of section 5.4 presents a complete analysis for the deformations of a tube under arbitrary loading. For the description of the warping it has been found convenient to introduce warping functions W_i* which, in each mode i, depend only on the lengthwise variation of the cross-section and loading. These functions simplify the discussion of specified displacement boundary conditions.

In practical problems it is often necessary to consider other than built-in or free conditions at the ends of the tube. To obtain the proper boundary conditions one has to apply combined systems of eigenloads at the ends of the tube. The properties of such end load systems are developed in 5.5.

The four-boom tube under arbitrary loading is the subject of section 5.6. Since the four-boom tube has but one eigenvalue λ_1 (see 4.3) the axial constraint stresses are obtained from the shear stress torque about the unique axis OS_1. Transverse loads through this axis

give stresses and deflections strictly in accordance with the ordinary engineers' theory. Thus in the four-boom tube the zero warping axis OS_1 takes properly the role often erroneously given to the flexural axis. Explicit formulæ for the position of the axis OS_1 are presented. In the special case when S_1 is at infinity there are no axial constraint stresses due to torque, but there will still be axial constraint stresses due to shear. The appropriate formulæ for the latter case are obtained as limiting forms of the general case; from these formulæ conditions are derived that a four-boom tube should have stresses strictly in accordance with the ordinary engineers' theory under *any* loading. Section 5.6. concludes with a comparison of the positions of the zero warping axis and flexural axis, as influenced by various parameters, in singly symmetrical four-boom rectangular and trapezoidal tubes. The results of this comparison are strikingly illustrated in Figs. 23 and 24. The authors would like to mention here a paper by Goodey, 1943[22] which investigated the singly symmetrical four-boom tube with nose cell under arbitrary loading. In this work, which came lately to the notice of the writers, Goodey correctly did not use the flexural axis to calculate the axial constraint stresses due to torsion, but used the total loading from the start. He treated the two spars as a statically determinate system and then determined, by strain energy methods ,the equal and opposite self-equilibrating load systems which must be applied to the spars and covers for compatibility with the rigid diaphragm assumption. Goodey's analysis is somewhat vitiated by his neglect of the shear deformation of the spars which may have a considerable effect, especially in modern wings.

It was stated in the summary and in 1.2 of the introduction that the six-boom tube would be analysed in Part 5. Since making this statement the writers realised that the method they developed for the six-boom tube could be applied more generally to the *n*-boom tube. Section 5.7 gives the complete theory for the single-cell *n*-boom tube of arbitrary, singly symmetrical, and doubly symmetrical cross-sections. The secular equation for the eigenvalues λ_i, the eigenfunctions h_i, the *H*-functions, and the co-ordinates ξ_i, η_i of S_i are all given. An extension of the method of 4.3.2 reduces the multi-cell *n*-boom tube, of the type shown in Fig. 16, to dependence on the analysis for an equivalent single-cell tube. The numerical application of this section is facilitated by certain approximate methods which will be developed in Part 6b.

Part 5 concludes with a detailed analysis of the eigenfunctions for a singly symmetrical trapezoidal tube with direct stress carrying top and bottom covers. For the limiting case of a doubly symmetrical rectangular tube the final results take a much simplified form. The uniform cylindrical tube of the latter cross-section has been treated previously by Hemp, 1943[12] and Hildebrand, 1943[14].

ACKNOWLEDGMENT.

The authors wish to record their appreciation of the continued support and encouragement in the preparation of this paper, given them by Captain J. L. Pritchard and Dr. D. M. A. Leggett.

5.2. ADDITIONAL NOTATION.

$G\bar{X}$, $G\bar{Y}$	Principal axes of root cross-section.
$\bar{x}$, $\bar{y}$	Co-ordinates *w.r.t.* axes $\bar{X}$, $\bar{Y}$.
$\bar{x}' = \rho\bar{x}$	
$\bar{y}' = \rho\bar{y}$	
$x_{rs} = x_s - x_r$	
$y_{rs} = y_s - y_r$	

$\delta_{r,\,r+1} = \dfrac{s_{r,\,r+1}}{t_{r,\,r+1}}$

$\delta_{rs} = \sum_{p=r}^{s-1} \delta_{p,\,p+1}$

$|pq,\ rs|$ — Defined by equation (413).

$|pq,\ rs|_x, |pq,\ rs|_y$ — Defined by equation (390).

$|ab,\ cd,\ ef|$ — Defined by equation (399).

A_{abc} — Area of triangle formed by booms a, b, c.

$2a$ — Distance between spars in trapezoidal tube.

D — Defined by equation (486).

T_B — Torque due to shear stress only.

T_{BE} — Value of T_B about point E' where an arbitrary axis OE meets diaphragm.

T_{BS} — Value of T_B for the flexural axis OE_S.

T_{Bi} — Value of T_B for the axis OS_i.

T_{Bio} — Value of T_{Bi} at root.

$Q_x = S_x - \dfrac{M_y}{r}$ — Shear resultant in X direction due to shear stress only.

$Q_y = S_y - \dfrac{M_x}{r}$ — Shear resultant in Y direction due to shear stress only.

$\omega_x,\ \omega_y$ — Distributed loads in X and Y directions.

$F_B = g_B h_B$ — Stress function for Bredt-Batho stress distribution.

$F_E = F_x + F_y$ — Stress function for ordinary engineers' theory stresses.

$F_x = g_x h_x$ — Stress function for loads in direction $G\bar{X}$.

$F_y = g_y h_y$ — Stress function for loads in direction $G\bar{Y}$.

$g_x,\ g_y$ — Functions of r.

$h_x,\ h_y$ — Functions of s.

$\sigma_E,\ \tau_E$ — Direct and shear stresses due to ordinary engineers' theory.

$\sigma_x,\ \tau_x$ — Values of σ_E and τ_E for loads in $G\bar{X}$ direction.

$\sigma_y,\ \tau_y$ — Values of σ_E and τ_E for loads in $G\bar{Y}$ direction.

$I_x,\ I_y,\ I_{xy}$ — Moments and products of inertia at root cross-section.

$L_{ri},\ L_{ri}$ — Defined by equation (340).

$a_{xi} = \xi_i \oint_S \dfrac{h_i}{t}\, ds$

$a_{yi} = \eta_i \oint_S \dfrac{h_i}{t}\, ds$

$\Delta h_m = h_{m,\,m+1} - h_{m-1,\,m}$

W_i^* — Warping function defined by equation (362).

$\bar{W}_i^*$ — Warping function for unit end load system.

e_i — "Fourier-coefficient" for end load distribution.

S_k — "Fourier-coefficient" for distributed shear load.

Δ — Secular determinant.

Ω — Co-factor of secular determinant.

$\binom{n}{m}$ — Binomial coefficient

$\bar{\lambda} = \lambda \left(\dfrac{t_s}{t_{12}}\right)^{\frac{1}{2}}$

N — Normalising factor.

5.3. ADDITIONAL REFERENCES.

19 H. Ebner.
Zur Festigkeit von Schalen und Rohrholmfluegeln.
Luftfahrtforschung, Vol. 14, No. 4/5, April 1937.

20. E. H. Atkin.
Tapered Beams. Suggested Solutions for Some Typical Aircraft Cases.
Aircraft Engineering, Vol. 10, Nos. 117 and 118 November and December, 1938.

21. A. Pflueger.
Spannungsverteilung in Stabfoermigen Membrankegelschalen.
Z.A.M.M., Vol. 22, No. 2, April 1942.

22. W. J. Goodey.
Two Spar Wing Stress Analysis. The Mathematical Theory with Numerical Examples.
Aircraft Engineering, Vol. 15, Nos 167 and 168, January and February 1943.

5.4. GENERAL THEORY FOR AN ARBITRARY SHEAR DISTRIBUTION.

The analysis for a conical tube under torque was first made in 4.1. for the particular case of a concentrated torque on a tube with $t_s^* = t^* = \rho^n$. A particular stress function was found which satisfied all the elastic and static conditions except the boundary conditions at the root and at the loaded section. The boundary requirements were then met by applying a suitable system of the eigenloads developed in Part 3. A parallel analysis can be applied also in the case of a tube under concentrated shear. However, the analysis for an arbitrary torque distribution given in 4.2, provides a suitable introduction to the immediate consideration of the analysis for a tube under arbitrary shear distribution.

As a preliminary it is necessary to deduce stress functions which are in static equilibrium with the applied load. These stress functions play an analogous part for arbitrary loading to the part played by the functions $g_B h_B$ developed in Part 4 for pure torque. For a uniform cylindrical tube they represent the stress distribution given by the classical engineers' theory of bending: for non-uniform cylindrical or conical tubes they may be regarded as a generalisation of the engineers' theory.

5.4.1. *The statically equivalent stress system for arbitrary loading.*

In the analysis of Parts 1 to 4 the directions of the axes X, Y were arbitrary. For the present analysis it is convenient to take these directions parallel to the principal axes of the cross-section. (The necessary modifications when X, Y are not parallel to the principal axes are given in 5.4.15.) This step involves no loss of generality and it is evident that the assumption,

$$t_s' = t_s t_s^* \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

ensures that the centroids of the cross-sections lie on a straight line OG joining the apex O to the centroid G of the root cross-section. The principal axes at the root are denoted by $G\bar{X}$, $G\bar{Y}$ (see Fig. 17) and at other sections the principal axes lie in the planes $OG\bar{X}$, $OG\bar{Y}$. The co-ordinates with respect to the principal axes are denoted by,

$$\left.\begin{aligned} \bar{x}' &= \rho \bar{x} \\ \bar{y}' &= \rho \bar{y} \end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (264)$$

Note that in the cylindrical tube the centroids lie on a line parallel to the generators.

In the theory of Parts 1 to 4 the torque loads were invariably referred to the axis OZ which is normal to the diaphragms and passes through the apex of the cone. The use of this axis in conical tubes allows a certain simplification in forming the equilibrium equations for torque, (20) or (47), since the torque about the OZ axis due to direct stresses is zero. Now the limiting form for torque equilibrium, equation (47a), was obtained by allowing $r_0 \to \infty$ with C a fixed point and therefore applies only to the right cylindrical tube. Since in Part 4 the case of pure torque only was considered, the results for cylindrical

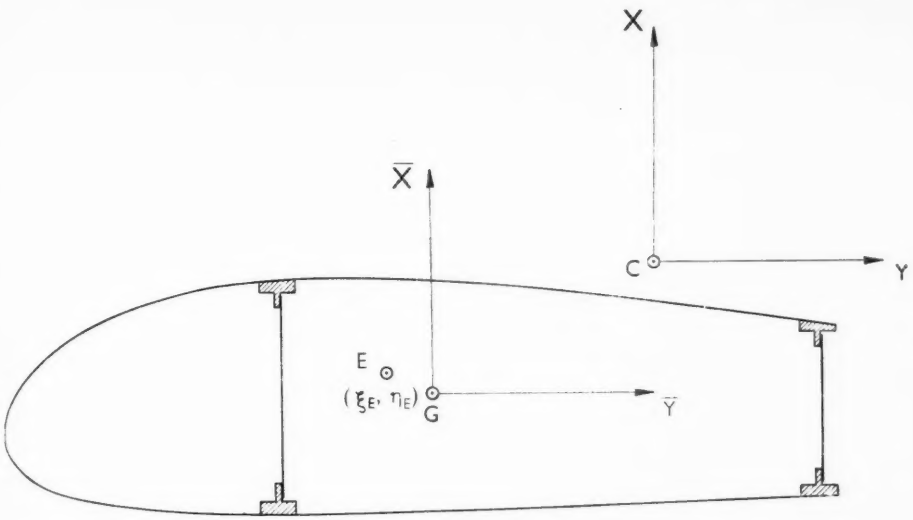


Fig. 17.
Arrangement of axes for arbitrary loading.

tubes there given do apply naturally to oblique cylindrical tubes, provided that the limitation stated in the summary is satisfied. However, in the general loading case the use of the axis OZ is inconvenient for obtaining the results for the swept-back cylindrical tube as limiting forms of those for the conical tube.

Accordingly the torque, which will be denoted by T_E, will be referred in the present analysis to an arbitrary axis OE where the point E in the root plane has the co-ordinates ξ_E, η_E. Note that in the conical tube the inclination of OE to OZ is arbitrary, but in the cylindrical tube OE is always parallel to the generators. It is important to note that the torque at any section is taken about the normal to the diaphragm at the point E' where OE meets the diaphragm.

The equilibrium conditions for an arbitrary load system in which the stresses are represented by a stress function F are,

$$\left.\begin{aligned}
-\rho\oint\frac{\partial F}{\partial r}\sin\psi\,ds &= S_x - \frac{M_y}{r}\\
-\rho\oint\frac{\partial F}{\partial r}\cos\psi\,ds &= S_y - \frac{M_x}{r}\\
\rho\oint\frac{\partial F}{\partial s}\,x\,ds &= M_y\\
\rho\oint\frac{\partial F}{\partial s}\,y\,ds &= M_x\\
-\rho^2\oint\frac{\partial F}{\partial r}\,p_E\,ds &= T_E - M_y\,m_E + M_x\,l_E
\end{aligned}\right\} \quad . \quad . \quad . \quad (265)$$

where l_E, m_E are the direction cosines of OE *w.r.t.* to axes OX and OY.

Note that,

$$l_E = \xi_E/r_o \text{ and } m_E = \eta_E/r_o \quad . \quad . \quad . \quad . \quad . \quad (266)$$

The limiting forms for a cylindrical tube become,

$$\left.\begin{aligned} -\oint \frac{\partial F}{\partial z'} \sin\psi \, ds &= S_x \\ -\oint \frac{\partial F}{\partial z'} \cos\psi \, ds &= S_y \\ \oint \frac{\partial F}{\partial s} x ds &= M_y \\ \oint \frac{\partial F}{\partial s} y ds &= M_x \\ -\oint \frac{\partial F}{\partial z'} p_E ds &= T_E - M_y m + M_x l \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (265a)$$

where l, m are the direction cosines of the generators *w.r.t.* axes OX, OY.

In the right cylindrical tube $l=m=0$ so that,

$$-\oint \frac{\partial F}{\partial z'} p_E ds = T_E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (265c)$$

The last equation in (265) and (265b) will be satisfied by a stress function,

$$F_B = g_B h_B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (267)$$

where h_B is the generalised Bredt-Batho function investigated in 4.1.2. Using equation (267) with equation (99) in the last of equations (265), one obtains for the conical tube,

$$\frac{dg_B}{dr} = -\frac{T_{BE}}{2A\rho^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (268)$$

where,

$$T_{BE} = T_E - M_y m_E + M_x l_E \quad . \quad . \quad . \quad . \quad . \quad . \quad (269)$$

and for the cylindrical tube,

$$\frac{dg_B}{dz'} = -\frac{T_{BE}}{2A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (268a)$$

where,

$$T_{BE} = T_E - M_y m + M_x l \quad . \quad . \quad . \quad . \quad . \quad . \quad (269a)$$

Thus T_{BE} is that part of the torque about E' resisted by shear flow in the tube. In the absence of bending moments equation (268) reduces to equation (144) of 4.2.1.

Note that the L.H.S. of the first four of equations (265) vanish for $F=F_B$ by virtue of the properties of h_B. Hence the complete stress function may be denoted by,

$$F = F_E + F_B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (270)$$

where F_E must satisfy the first four, and make zero the L.H.S. of the last, of equations (265), so that,

$$\oint \frac{\partial F_E}{\partial r} p_E ds = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (265b)$$

5.4.2. *The generalised engineers' theory stress systems*: $g_x h_x$, $g_y h_y$.

It will now be shown that the stress function F_E may be expressed as,

$$F_E = g_x h_x + g_y h_y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (271)$$

where the functions $g_x h_x$ and $g_y h_y$ correspond to the load systems S_x and S_y respectively, which act in each section through E' parallel to the principal axes $G\bar{X}$ and $G\bar{Y}$.

It is necessary only to investigate in detail the stress system due to S_x since the corresponding system for S_y follows immediately. The bending moment associated with S_x is M_y which acts about the axis $G'\bar{Y}'$. M_x and M_y are taken positive when giving rise to tension stresses (positive σ) for positive $\bar{y}$ and $\bar{x}$ respectively.

With the assumption $n \approx 1$ the direct stress σ_x due to M_y is given by,

$$\sigma_x = \frac{M_y \bar{x}'}{I_y'} = \frac{M_y \bar{x}}{\rho^2 t_s^* I_y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (272)$$

The stress σ_x will be assumed to derive from the stress-function,

$$F_x = g_x h_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (273)$$

Then from equations (24) and (28)

$$\sigma_x = \frac{1}{\rho t_s^*} g_x \left(\frac{1}{t_s} \frac{dh_x}{ds} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (274)$$

Comparing equations (272) and (274) it will be seen that suitable forms for g_x and h_x are

$$\left.\begin{aligned} g_x &= \frac{M_y}{\rho S} \\ h_x &= \frac{S}{I_y} \int^s t_s \bar{x} ds \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (275)$$

where S is the periphery at the root section. Note that the form chosen for h_x is non-dimensional so as to be consistent with previous h-functions, like h_i, h_B etc. The undetermined constant in the integral

$$\int^s t_s \bar{x} ds$$

will be determined by the conditions given in equation (265b).

From equations (24) and (28) the shear stress τ_x may be written,

$$\tau_x = -\frac{1}{t^*} \frac{dg_x}{dr} \frac{h_x}{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (276)$$

With equation (275)

$$\frac{dg_x}{dr} = \frac{Q_x}{\rho S} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (277)$$

where

$$Q_x = \left(S_x - \frac{M_y}{r} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (278)$$

represents the proportion of the shear resultant resisted by shear stresses, while (M_y/r) is the shear load resisted by the direct stresses. These results may be compared with previous investigations of Pflueger, 1942[21] on the engineering stresses in conical tubes and Atkin, 1938[20], Cox, 1942[5] on tapered beams.

Equation (276) can now be written,

$$\left.\begin{aligned} \tau_x &= -\frac{Q_x}{\rho t^*} \frac{h_x}{tS} \\ \text{or} \qquad \tau_x &= -\frac{Q_x}{\rho t^*} \frac{\int^s t_s \bar{x} ds}{t I_y} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (276a)$$

which reduces for a cylindrical tube to,

$$\tau_x = -\frac{S_x}{t^*} \frac{\int^s t_s \bar{x} ds}{t I_y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (276b)$$

which with $t^* = 1$ becomes the well-known formula for the engineers' theory shear stresses in a uniform cylindrical tube.

The corresponding stress function

$$F_y = g_y h_y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (273a)$$

for loads in the Y direction is given by,

$$\left.\begin{aligned} g_Y &= \frac{M_x}{\rho S} \\ h_Y &= \frac{S}{I_x}\int t_s \bar{y} ds \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (275a)$$

Note that the undetermined constant in the integral

$$\int t_s \bar{y} ds$$

is in general different from that in the integral of equation (268).

The equations corresponding to (277) and (278) for S_y are

$$\frac{dg_y}{dr} = \frac{Q_y}{\rho S} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (277a)$$

where,

$$Q_y = \left(S_y - \frac{M_x}{r}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (278a)$$

Note also that the distributed loadings ω_x and ω_y acting in the directions XO and YO are related to S_x, Q_x and S_y, Q_y respectively as follows,

$$\frac{d}{dr}S_x = \frac{1}{r}\frac{d}{dr}(rQ_x) = \omega_x$$

$$\frac{d}{dr}S_y = \frac{1}{r}\frac{d}{dr}(rQ_y) = \omega_y$$

Equations (275) and (275a) prove that the stress function F_E may be expressed as written in equation (271).

The total engineers' theory stresses will be denoted by σ_E and τ_E, so that,

$$\left.\begin{aligned} \sigma_E &= \sigma_x + \sigma_Y \\ \tau_E &= \tau_x + \tau_Y \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (279)$$

From equation (276a),

$$Q_x = \rho\oint \tau_x t^* t \sin\psi ds = -\frac{Q_x}{S}\oint h_x \sin\psi ds$$

or

$$\oint h_x \sin\psi ds = -S \quad . \quad . \quad . \quad . \quad . \quad (280a)$$

and similarly,

$$\oint h_Y \cos\psi ds = -S \quad . \quad . \quad . \quad . \quad . \quad (280b)$$

Note also

$$\oint h_x \cos\psi ds = \oint h_Y \sin\psi ds = 0 \quad . \quad . \quad . \quad . \quad . \quad (281)$$

Since $F_E = F_x + F_Y$ must satisfy equation (265b) independently of the loading functions g_x and g_Y, it follows that,

$$\oint h_x p_E ds = \oint h_Y p_E ds = 0 \quad . \quad . \quad . \quad . \quad . \quad (282)$$

These conditions will be applied in 5.4.3. where the actual h_x and h_Y distributions are obtained for single and multi-cell tubes.

Some important relations connecting the functions h_x, h_i and h_B will now be derived. The first relation to be investigated is,

$$\oint \frac{h_i h_x}{t} ds$$

Substituting for $\frac{h_i}{t}$ from equation (45) and using the relation

$$p = p_E + \xi_E \cos\psi - \eta_E \sin\psi \quad . \quad . \quad . \quad . \quad . \quad . \quad (283)$$

one obtains

$$\oint \frac{h_i h_x}{t}\,ds = -\frac{1}{\lambda_i^2}\oint h_x \frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)ds + \frac{1}{2A}\oint_S \frac{h_i}{t}\,ds\Big[\oint h_x p_E ds - (\xi_i - \xi_E)\oint h_x \cos\psi\,ds + (\eta_i - \eta_E)\oint h_x \sin\psi\,ds\Big] \quad . \quad . \quad (A)$$

By partial integration of the first integral on the R.H.S. and referring to the multi-cell tube of Fig. 4, p. 222, one obtains,

$$\oint h_x \frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)ds = \sum_{m=1}^{n}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)_m \left\{B_m\left(\frac{1}{t_s}\frac{dh_x}{ds}\right)_m - h_{xm+} - h_{xmd} + h_{xm-}\right\} - \oint \frac{1}{t_s}\frac{dh_x}{ds}\frac{dh_i}{ds}\,ds \quad (B)$$

where the suffix m refers to the mth joint in a tube with n joints and the suffices $+$, $-$, and d have the same meaning as in 3.2.1.. By virtue of equations of the type (63) the expression in the wavy brackets is zero; the same result will be true for any multi-cell tube. Consider now the remaining term on the R.H.S. of equation B. From the second expression in equations (275) this term can be written,

$$\frac{S}{I_y}\oint \bar{x}\frac{dh_i}{ds}\,ds = 0$$

since h_i is a self-equilibrating system (see equations (52)). Hence the first term on the R.H.S. of equation (A) is zero. Thus equation (A), using equations (280) to (282), reduces to,

$$\oint \frac{h_i h_x}{t}\,ds = -\frac{(\eta_i - \eta_E)S}{2A}\oint_S \frac{h_i}{t}\,ds \quad . \quad . \quad . \quad . \quad . \quad (284a)$$

Similarly,

$$\oint \frac{h_i h_y}{t}\,ds = +\frac{(\xi_i - \xi_E)S}{2A}\oint_S \frac{h_i}{t}\,ds \quad . \quad . \quad . \quad . \quad . \quad (284b)$$

It will now be shown that,

$$\oint \frac{h_B h}{t}\,ds = \oint_S \frac{h}{t}\,ds \quad . \quad . \quad . \quad . \quad . \quad . \quad (285)$$

where the form of h is restricted only by the requirements of equal twist in all cells. This result has been given for the particular case where $h = h_i$ in equation (118).

The h_B distribution in an N-cell tube may be represented by N clockwise distributions of the form h_{BM} in the Mth cell. Thus in the outer walls

$$h_B = h_{BM}$$

and in the wall between two cells M and M'

$$h_B = h_{BM} - h_{BM'}$$

if the positive s direction is clockwise in the cell M.

Now using these relations,

$$\oint \frac{h_B h}{t}\,ds = \sum_{M=1}^{N} h_{BM}\oint_M \frac{h}{t}\,ds = \sum_{M=1}^{N} 2A_M h_{BM}\left(\frac{1}{2A_M}\oint_M \frac{h}{t}\,ds\right)$$

But

$$\frac{1}{2A_M}\oint_M \frac{h}{t}\,ds = \frac{1}{2A}\oint_S \frac{h}{t}\,ds$$

and $\sum_{M=1}^{N} 2A_M h_{BM}$ = total "torque" due to the h_B distribution $= 2A$ (see equation (99)).

Equation (285) follows immediately. In particular,

$$\oint \frac{h_B^2}{t}\,ds = \oint_S \frac{h_B}{t}\,ds \qquad (285a)$$

$$\oint \frac{h_B h_X}{t}\,ds = \oint_S \frac{h_X}{t}\,ds \qquad (285b)$$

$$\oint \frac{h_B h_Y}{t}\,ds = \oint_S \frac{h_Y}{t}\,ds \qquad (285c)$$

5.4.3. *Determination of the h_X and h_Y functions for single and multi-cell tubes.* Consider first the single-cell tube with the cross-section illustrated in Fig. 18.

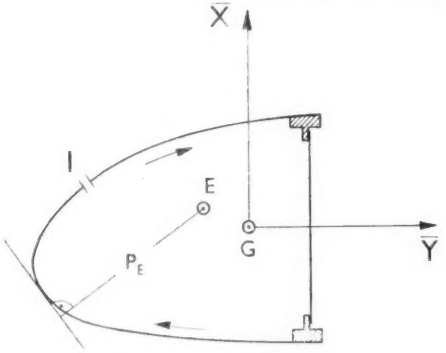


Fig. 18.
Determination of h_x-distribution in a single cell tube.

Assuming the section to be cut at point *I* the function h_{xo} for the resulting open section is statically determined since the origin of the integral (275) is at the point *I*. The distribution h_x of the actual tube is given by,

$$h_x = h_{xo} + h_{xI} \qquad (286)$$

where h_{xI} is the unknown value of h_x at point *I*.

Using equation (286) in (282),

$$h_{xI} = -\frac{1}{2A}\oint h_{xo}\,p_E ds \qquad (287)$$

which with equation (286) determines completely the function h_x. A similar analysis holds for h_y.

For a multi-cell tube of the type illustrated in Fig. 19 each cell is supposed cut at points I, II, etc., in the outer walls of each cell I, II, etc. The h_{xo} system for the resulting open tube may easily be obtained by integration from each cut with due attention to the sign of s. The positive direction of s may be assigned arbitrarily in each wall but once assigned must be adhered to strictly. In Fig. 19 the positive directions of s have been denoted by arrows. The closed N-cell tube will have N unknowns

$$h_{xI}, h_{xII}, \ldots\ldots, h_{xN}$$

at the cuts,

$$I, II, \ldots\ldots N.$$

Now there are $N-1$ conditions for the equality of twist of each cell and the one condition given by equation (282), in all N conditions for the N unknowns. The h_x distribution

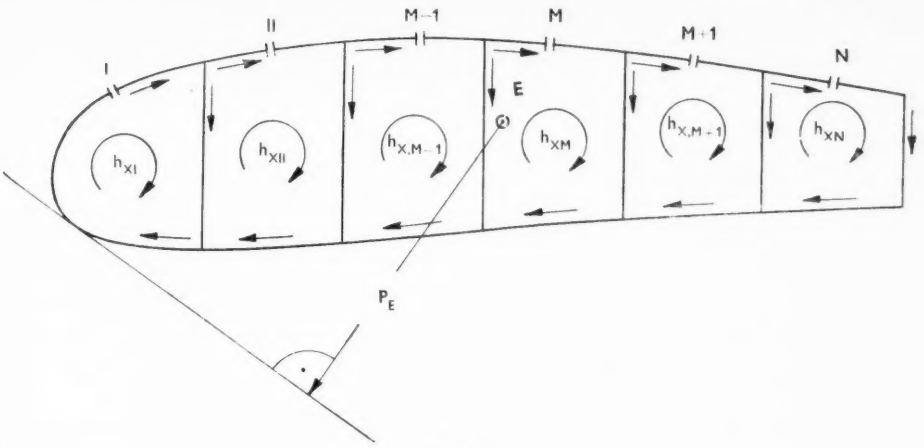


Fig. 19.
Determination of h_x-distribution in a multi-cell tube.

in the actual system can now be described as follows:

In the outer walls of the *M*th cell,

$$h_x = h_{xo} + h_{xM} \quad . \quad . \quad . \quad . \quad . \quad . \quad (288a)$$

in the left hand wall of the *M*th cell,

$$h_x = h_{xo} + h_{x(M-1)} - h_{xM} \quad . \quad . \quad . \quad . \quad . \quad . \quad (288b)$$

and in the right hand wall of the *M*th cell,

$$h_x = h_{xo} + h_{xM} - h_{x(M+1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (288c)$$

The rate of twist of the *M*th cell is proportional to,

$$C_x = \frac{1}{A_M} \oint_M \frac{h_x}{t}\, ds \quad . \quad . \quad . \quad . \quad . \quad . \quad (289)$$

where,

$$\oint_M \frac{h_x}{t}\, ds = \oint_M \frac{h_{xo}}{t}\, ds - h_{x(M-1)} \int_{M-1,\,M} \frac{ds}{t} + h_{xM} \oint_M \frac{ds}{t} - h_{x(M+1)} \int_{M,\,M+1} \frac{ds}{t} \quad . \quad . \quad (289a)$$

The integral $\int_{M-1,\,M}$ refers to the integration over the wall between the $(M-1)$th and *M*th cells. Care must be taken in regard to the signs of the integrals in equation (289a). In all cases the sign of the element *ds* must be taken as positive *even when the clockwise sense about the Mth cell opposes the assigned positive direction of s* (see Fig. 19).

Thus all the three coefficients,

$$\int_{M-1,\,M} \frac{ds}{t}, \oint_M \frac{ds}{t}, \int_{M,\,M+1} \frac{ds}{t}$$

are positive and in the integral

$$\oint_M \frac{h_{xo}}{t}\, ds$$

the sign of h_{xo} in the wall $M-1$, *M* has to be reversed, *i.e.* for that part of the integral take,

$$-\int_{M-1,\,M} \frac{h_{xo}}{t}\, ds$$

Since the rate of twist is the same for all cells one obtains from equations (289) and (289a) N equations:

$$\left.\begin{aligned}
&-h_{x\mathrm{I}}\oint_{\mathrm{I}}\frac{ds}{t}-h_{x\mathrm{II}}\int_{\mathrm{I,\,II}}\frac{ds}{t}=A_1C_x-\oint_{\mathrm{I}}\frac{h_{x0}}{t}\,ds\\
&\text{---------------------------------}\\
&-h_{x(M-1)}\int_{M-1,\,M}\frac{ds}{t}+h_{xM}\oint_{M}\frac{ds}{t}-h_{x(M+1)}\int_{M,\,M+1}\frac{ds}{t}=A_MC_x-\oint_{M}\frac{h_x}{t}\,ds\\
&\text{---------------------------------}\\
&h_{x(N-1)}\int_{N-1,\,N}\frac{ds}{t}+h_{xN}\oint_{N}\frac{ds}{t}=A_NC_x-\oint_{N}\frac{h_{x0}}{t}\,ds
\end{aligned}\right\}\quad .\quad (290)$$

Equations (290) are solved for h_{xM} and the solution can be written in the form,

$$h_{xM}=\sum_{R=1}^{N}a_{M,\,R}\left(A_RC_x-\oint_{M}\frac{h_{x0}}{t}\,ds\right)=C_x\sum_{R=1}^{N}a_{M,\,R}\,A_R-\sum_{R=1}^{N}a_{M,\,R}\oint_{M}\frac{h_{x0}}{t}\,ds\quad .\quad (291)$$

The constant C_x is determined from equation (282). Substituting equations (288) into the latter,

$$\oint p_Eh_{x0}ds+\sum_{M=1}^{N}2A_Mh_{xM}=0\quad .\quad .\quad .\quad .\quad .\quad .\quad (292)$$

The constant C_x is now obtained by using equations (291) in (292).

A similar investigation gives the h_y distribution in which the coefficients $a_{M,\,R}$ will be the same but C_x will be replaced by C_y.

The essentials of the above analysis have been given by Ebner, 1937[19] for uniform right cylindrical tubes, and Pflueger, 1942[21] mentioned the possibility of extending the theory to conical tubes. Ebner referred the torque to the shear centre (*i.e.* he took $C_x=C_y=0$). This may be done, if desired, by taking $C_x=0$ in equations (291) when h_{xM} can be directly calculated. Using now relation (283) in (292) one obtains Ebner's results for the co-ordinates η_{ES} and ξ_{ES} of the shear centre E_S from the formulæ

$$\eta_{ES}S=\oint ph_{x0}ds-\sum_{M=1}^{N}2A_Mh_{xM}\quad .\quad .\quad .\quad .\quad .\quad (293a)$$

and similarly,

$$\xi_{ES}S=-\oint ph_{y0}ds-\sum_{M=1}^{N}2A_Mh_{yM}\quad .\quad .\quad .\quad .\quad (293b)$$

If the axis OE_S is defined by conditions (293) then zero rate of twist for the statically equivalent stress system is obtained throughout the tube if,

$$T_{BS}=0\quad .\quad .\quad .\quad .\quad .\quad .\quad (269a)$$

where,

$$T_{BS}=T-M_ym_{ES}+M_xl_{ES}\quad .\quad .\quad .\quad .\quad .\quad (269b)$$

This condition will be satisfied throughout the tube *if the loading on the tube consists of transverse loads all passing through the axis* OE_S. Note that the above arguments apply under the assumption that the stresses are distributed according to ordinary engineers' theory. In particular a concentrated load S_x acting at any point E_S' of the axis OE_S will satisfy (269a) and hence produce zero twist.

This problem may be discussed from a slightly different angle. If the transverse loads pass through any axis OE, then since the direct stress shear resultant passes through O the internal shear stress resultants Q_x and Q_y must at any section act through OE. It follows that if E coincides with E_S there will be no twist at any section. It follows further that the rate of twist under any loading will be given by,

$$\frac{d\theta}{dr} = \frac{T_{BS}}{4A'^2G}\oint_S \frac{h_B}{t}\,ds' = \frac{T_{BS}}{4A^2G\rho^3t^*}\oint_S \frac{h_B}{t}\,ds \quad . \quad . \quad . \quad (294)$$

where T_{BS} is defined by equation (269b).

Note that if the tube is right cylindrical or more generally if OC coincides with OE_S, or if $M_x = M_y = 0$,

$$T_{BS} = T$$

so that in these cases equation (294) reduces to the well-known Bredt-Batho expression for rate of twist.

It will appear from the foregoing remarks that the shear centre is still a useful conception when considering the overall deformations of swept-back cylindrical or conical tubes due to the statically equivalent stress system $F_B + F_E$.

5.4.4. *The differential equation in the g_i-function for arbitrary load distribution.*

Having now analysed completely a statically equivalent stress system

$$F = F_B + F_E \quad . \quad . \quad . \quad . \quad . \quad . \quad (270)$$

in 4.1.2, 5.4.1 and 5.4.2-3 the analysis will be completed by assuming the total stress function to be of the form,

$$F = \sum_i g_i h_i + F_B + F_E$$

or

$$F = \sum_i g_i h_i + g_B h_B + g_x h_x + g_y h_y \quad . \quad . \quad . \quad . \quad . \quad (295)$$

which is analogous to equation (143) of 4.2.1 for the case of an arbitrary torque distribution T.

From 3.1 the differential equation for the stress function F is,

$$\frac{Gt^*}{Et_s^*}\frac{\partial}{\partial s}\left(\frac{1}{t_s}\frac{\partial F}{\partial s}\right) + \frac{\rho^2}{t}\frac{\partial^2 F}{\partial r^2} + \frac{t^*}{t}\rho\frac{d}{dr}\left(\frac{\rho}{t^*}\right)\frac{\partial F}{\partial r}$$
$$= -Gt^*\rho\left[p\frac{d}{dr}\left(\rho^2\frac{d\theta}{dr}\right) + \rho\sin\psi\frac{d^2u}{dr^2} + \rho\cos\psi\frac{d^2v}{dr^2}\right] \quad . \quad (27a)$$

which with equation (295) and division by ρt^* becomes,

$$\sum_i\left\{\frac{h_i}{t}\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_i}{dr}\right) + \frac{G}{E}\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)\frac{g_i}{\rho t_s^*}\right\}$$
$$+ \frac{h_B}{t}\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_B}{dr}\right)$$
$$+ \frac{h_x}{t}\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_x}{dr}\right) + \frac{G}{E}\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_x}{ds}\right)\frac{g_x}{\rho t_s^*}$$
$$+ \frac{h_y}{t}\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_y}{dr}\right) + \frac{G}{E}\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_y}{ds}\right)\frac{g_y}{\rho t_s^*}$$
$$= -G\left[p\frac{d}{dr}\left(\rho^2\frac{d\theta}{dr}\right) + \rho\sin\psi\frac{d^2u}{dr^2} + \rho\cos\psi\frac{d^2v}{dr^2}\right] \quad . \quad . \quad (296)$$

Now from equations (275) and (275a)

$$\left.\begin{aligned}\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_x}{ds}\right) &= \frac{S}{I_y}\sin\psi\\ \frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_y}{ds}\right) &= \frac{S}{I_x}\cos\psi\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (297)$$

Multiplying equation (296) by h_i, integrating over all walls and using the orthogonality and normalising conditions (76), (77), (57) and (78) and the relations expressed by equations (54), (118), (284a), (284b) and (297) one obtains,

$$\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_i}{dr}\right) - \mu_i^2\,\frac{g_i}{\rho t_s^*} + \oint_S \frac{h_1}{t}\,ds\left\{\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_B}{dr}\right) - \frac{(\eta_i-\eta_E)S}{2A}\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_x}{dr}\right) + \frac{(\xi_i-\xi_E)S}{2A}\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_y}{dr}\right)\right\} = 0 \quad . \qquad (298)$$

Substituting for $\frac{dg_B}{dr}$, $\frac{dg_x}{dr}$, and $\frac{dg_y}{dr}$ from equations (268), (277) and (277a) equation (298) becomes,

$$\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_i}{dr}\right) - \mu_i^2\,\frac{g_i}{\rho t_s^*} = +\frac{1}{2A}\oint_S \frac{h_1}{t}\,ds\cdot\frac{d}{dr}\left(\frac{T_{Bi}}{\rho t^*}\right) \quad . \qquad (299)$$

where

$$T_{Bi} = T_{BE} + \rho\,(\eta_i - \eta_E)Q_x - \rho\,(\xi_i - \xi_E)\,Q_y \quad . \quad . \quad . \quad . \quad (300)$$

For a cylindrical tube equations (299) and (300) reduce to,

$$\frac{d}{dz'}\left(\frac{1}{t^*}\frac{dg_i}{dz'}\right) - \mu_i^2\,\frac{g_i}{t_s^*} = +\frac{1}{2A}\oint_S \frac{h_1}{t}\,ds\cdot\frac{d}{dz'}\left(\frac{T_{Bi}}{t^*}\right) \quad . \quad . \quad . \quad (299a)$$

where

$$T_{Bi} = T_{BE} + (\eta_i - \eta_E)\,S_x - (\xi_i - \xi_E)\,S_y \quad . \quad . \quad . \quad . \quad . \quad (300a)$$

Equations (299) may also be written,

$$r^2\frac{dg_i}{dr} + r\left(1 - \frac{r}{t^*}\frac{dt^*}{dr}\right)\frac{dg_i}{dr} - \mu_i^2 r_o^2\,\frac{t^*}{t_s^*}\,g_i = -\frac{r_o^2}{2A}\oint_S \frac{h_1}{t}\,ds\cdot\left\{\frac{T_{Bi}}{r}\left(1 + \frac{r}{t^*}\frac{dt^*}{dr}\right) - \frac{dT_{Bi}}{dr}\right\} \qquad (301)$$

for a conical tube,

and

$$\frac{d^2g_i}{dz'^2} - \frac{1}{t^*}\frac{dt^*}{dz'}\frac{dg_i}{dz'} - \mu_i^2\,\frac{t^*}{t_s^*}\,g_i = -\frac{1}{2A}\oint_S \frac{h_1}{t}\,ds\cdot\left\{\frac{1}{t^*}\frac{dt^*}{dz'}T_{Bi} - \frac{dT_{Bi}}{dz'}\right\} \quad . \quad . \qquad (301a)$$

for a cylindrical tube.

Note that T_{Bi} is the torque, due only to the shear flow in the tube, about the points S_i' where OS_i meets the diaphragms. Hence equations (300) and (300a) may be written in the forms,

$$T_{Bi} = T_i - M_y m_i + M_x l_i, \text{ for conical tubes} \quad . \quad . \quad . \quad . \quad (302)$$

and

$$T_{Bi} = T_i - M_y m + M_x l, \text{ for cylindrical tubes} \quad . \quad . \quad . \quad (302a)$$

which are the same as equations (269) with the point $S_i\,(\xi_i, \eta_i)$ in place of E.

Formulæ (300) and (300a) may also be written,

$$T_{Bi} = T + \rho\eta_i Q_x - \rho\xi_i Q_y, \text{ for conical tubes} \quad . \quad . \quad . \quad . \quad (300b)$$

and

$$T_{Bi} = T + \eta_i S_x - \xi_i S_y, \text{ for right cylindrical tubes} \quad . \quad . \quad . \quad (300c)$$

Comparing equations (301) with equations (146) one sees that the *differential equation determining g_i for any loading is exactly similar to that for pure torque except that T is replaced by T_{Bi}. Thus the g_i function for any loading is obtained by considering only the torque T_{Bi} about S_i'.*

For an arbitrary distribution of T_{Bi}, t^* and/or t_s^* the solution of equations (299) or (301) may be found by numerical methods. When t^*, t_s^* and T_{Bi} and their derivatives are continuous over a section of the tube the solution of equation (299) over this section is the sum of the complementary function (79) involving the two undetermined constants D_{1i} and D_{2i} and a particular solution. An interesting special case occurs when,

$$\left.\begin{aligned} T_{Bi} &= T_{Bio}\rho t^* \text{ for conical tubes} \\ T_{Bi} &= T_{Bio}t^* \text{ for cylindrical tubes} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (303)$$

Then the R.H.S. of equation (299) is zero and so also is the particular solution. Hence for the T_{Bi} distribution of (303) the stresses are distributed according to the ordinary engineers' theory provided that the ends of the tube are free to warp. The physical interpretation is that for the loadings of equations (303) the warping is independent of r.

From equation (300b) it is seen that for a load

$$\left.\begin{aligned} Q_x &= Q_{xo}t^* \\ Q_y &= Q_{yo}t^* \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (304)$$

through the Z-axis ($T=0$) condition (303) is satisfied. The distributed load associated with condition (304) is,

$$\left.\begin{aligned} \omega_x &= \frac{Q_{xo}}{r}\frac{d}{dr}(rt^*) \\ \omega_y &= \frac{Q_{yo}}{r}\frac{d}{dr}(rt^*) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (305)$$

If the load corresponding to equation (304) acts through any line connecting the apex O with a point $E(\xi_E, \eta_E)$ the torque T is given by,

$$T = (Q_{yo}\xi_E - Q_{xo}\eta_E)\rho t^* = T_o\rho t^* \quad . \quad . \quad . \quad . \quad . \quad (306)$$

and condition (303) is again satisfied.

For the right cylindrical tube condition (304) becomes,

$$\left.\begin{aligned} S_x &= S_{xo}t^* \\ S_y &= S_{yo}t^* \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (304a)$$

The distributed load associated with (304a) is

$$\left.\begin{aligned} \omega_x &= S_{xo}\frac{dt^*}{dz'} \\ \omega_y &= S_{yo}\frac{dt^*}{dz'} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (305a)$$

The loads corresponding to (304a) may act through any line (ξ_E, η_E) parallel to the generator, since equation (300a) then reduces to,

$$T_{Bi} = T_i = [S_{yo}(\xi_E - \xi_i) - S_{xo}(\eta_E - \eta_i)]t^* = T_{io}t^* \quad . \quad . \quad . \quad (307)$$

For the swept-back cylindrical tube it is in general impossible to satisfy condition (303) in the presence of shear loads. For if T_{Bi} satisfies condition (303) for a particular value of i, from (302a) T_i, M_y and M_x are all proportional to t^*. But

$$T_{Bj} = T_{Bi} - S_x(\eta_j - \eta_j) + S_y(\xi_i - \xi_j)$$

so that if $T_{Bj} \propto t^*$

$$S_x, S_y \propto t^*$$

Thus the moments and shears are both proportional to t^* which is only possible if

$$t^* \propto e^{z'}.$$

Hence in general in a swept-back tube engineers' theory stresses are not compatible in the presence of shear loads.

Note that in a conical tube loaded at the tip

$$Q_x = Q_{xo} = Q_y = Q_{yo} = T = 0$$

so that the conditions of equations (304) and (306) are fulfilled for any t^*. Also in this case there is no shear stress and hence no warping. Thus for a built-in conical tube with a load at the apex the direct stresses are given correctly by the ordinary engineers' theory, and the shear force at any cross section is sustained by the component of the direct stresses in the plane of the diaphragms.

In general the loading will not fall under one of the special cases just considered and the stresses will take the form,

$$\left.\begin{aligned}\sigma &= \sigma_E + \frac{1}{\rho t_s^*}\sum_i g_i\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)\\ \tau &= \tau_B + \tau_E - \frac{1}{t^*}\sum_i\left(\frac{dg_i}{dr}\frac{h_i}{t}\right)\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (308)$$

The undetermined constants D_{1i} and D_{2i} in the g_i-function are obtained from the boundary conditions for the stresses and/or warping, which will now be considered. To derive the boundary conditions associated with warping the expression for $\frac{\partial w}{\partial s}$ as given in equation (26) will be used. The reader is also referred to 5.4.16 where the discussion of the warping is facilitated by the introduction of the warping function W_i^*, analogous to W^* for a four-boom tube.

Using F, as in equation (295), in (26), one obtains with $t' = tt^*$,

$$\frac{\partial w}{\partial s} = -\frac{1}{Gt^*}\rho\left[\frac{dg_B}{dr}\frac{h_B}{t} + \frac{dg_x}{dr}\frac{h_x}{t} + \frac{dg_y}{dr}\frac{h_y}{t} + \sum_i\frac{dg_i}{dr}\frac{h_i}{t}\right] - p\rho^2\frac{d\theta}{dr} - \rho\frac{du}{dr}\sin\psi - \rho\frac{dv}{dr}\cos\psi \quad . \quad . \quad . \quad . \quad (309)$$

For the cylindrical tube put $\rho = 1$ and $r = z'$.

At a free end $r = r_e$, $\sigma = \sigma_E = 0$ so that from equation (308),

$$\sum_i g_i\left(\frac{1}{t_s}\frac{dh_i}{ds}\right) = 0$$

Multiplying by $\frac{dh_i}{ds}$, integrating over all walls and using equation (77),

$$[g_i]_{r=r_e} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (310)$$

At a built-in end, $r = r_o$, the boundary conditions are most conveniently taken in the form, $\frac{\partial w}{\partial s} = 0$, so that from equation (309) multiplying by h_i, integrating over all walls and using (54), (57), (76), (118), (268), (275) and (284),

$$\left[\frac{dg_i}{dr}\right]_{r=r_o} = \frac{1}{2A}\left[T_{BEo} + Q_{xo}(\eta_i - \eta_E) - Q_{yo}(\xi_i - \xi_E)\right]\oint_s\frac{h_i}{t}\,ds$$

or with equation (300),

$$\left[\frac{dg_i}{dr}\right]_{r=r_o} = \frac{T_{Bio}}{2A}\oint_s\frac{h_i}{t}\,ds \quad . \quad . \quad . \quad . \quad . \quad . \quad (311)$$

If there are no discontinuities in load or structure between $r = r_e$ and $r = r_o$ the two conditions (310) and (311) are sufficient to determine the coefficients D_{1i}, D_{2i} in the complementary function which holds over the whole region.

Each discontinuity in T_{Bi}, t^* and/or t_s^* requires two additional constants D which can be found from the two additional boundary conditions at the discontinuity.

The general case of a discontinuity in T_{Bi}, t^* and/or t_s^* will next be considered. Denoting the two sides of the discontinuity at $r = r_1$ by the suffices R and L, one obtains from the equality of direct stress flow,

$$\sum_i [g_{iR}]_{r=r_1}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right) = \sum_i [g_{iL}]_{r=r_1}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)$$

and hence by the procedure used to establish equation (310),

$$[g_{iR}]_{r=r_1} - [g_{iL}]_{r=r_1} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (312)$$

From the equality of warping on each side of the discontinuity, application of equation (309), and a procedure similar to that establishing equation (311) yields,

$$\frac{1}{t_R^*}\left[\frac{dg_{iR}}{dr}\right]_{r=r_1} - \frac{1}{t_L^*}\left[\frac{dg_{iL}}{dr}\right]_{r=r_1} = \frac{1}{2A\rho_1^2}\left\{\frac{T_{BiR}}{t_R^*} - \frac{T_{BiL}}{t_L^*}\right\}\oint_S \frac{h_i}{t}\,ds \quad . \quad . \quad (313)$$

where T_{BiR} and T_{BiL} are the values of T_{Bi} on each side of $r=r_1$.

For a cylindrical tube equations (310) to (313) become,

$$[g_i]_{z'=0} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (310a)$$

$$\left[\frac{dg_i}{dz'}\right]_{z'=c} = \frac{T_{Bio}}{2A}\oint_S \frac{h_i}{t}\,ds \quad . \quad . \quad . \quad . \quad (311a)$$

$$[g_{iR}]_{z'=z_1} - [g_{iL}]_{z'=z_1} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (312a)$$

$$\frac{1}{t_R^*}\left[\frac{dg_{iR}}{dz'}\right]_{z'=z_1} - \frac{1}{t_L^*}\left[\frac{dg_{iL}}{dz'}\right]_{z'=z_1} = \frac{1}{2A}\left\{\frac{T_{BiR}}{t_R^*} - \frac{T_{BiL}}{t_L^*}\right\}\oint_S \frac{h_i}{t}\,ds \quad . \quad . \quad (313a)$$

Comparing equations (310) to (313) with equations (151) to (154) one sees that the boundary conditions for g_i in the general loading case are exactly similar to those for pure torque except that T is replaced by T_{Bi}. Thus for the calculation of the axial constraint stresses in the general loading case it is only necessary to consider in each mode i the torque T_{Bi} about S_i'. This is an important result of the present theory showing a remarkable unification of the analysis for the general loading case with that for pure torque. It now appears that the axial constraint stresses may be regarded as corrections due to torque only, but to each eigenvalue λ_i there corresponds a different axis OS_i about which the torque T_{Bi} is calculated. A common practice is to calculate the torque, from which approximate axial constraint stresses are found, about the shear centre axis OE_S. That this procedure is in general incorrect may be seen immediately from the fact that in the four-boom tube there is but one axis OS_1 corresponding to the only eigenvalue λ_1 so that the exactly correct procedure in this case is to refer the torque to the axis OS_1 in general different from OE_S. How greatly the positions of S_1 and E_S may differ is shown clearly in 5.6.2 where the particular case of a singly symmetrical trapezoidal tube with four booms is investigated. It is also evident that a first approximation to the axial constraint stresses in a tube with more than one eigenvalue λ_i is more logically obtained by referring the torque to the axis OS_1 corresponding to the lowest eigenvalue λ_1 than to the shear centre.

It was stated in the introduction (1.2, p. 204) that the stresses due to any shear load distribution could be expressed in a similar form to equations (1). In fact from equations (308) one may write,

$$\left.\begin{aligned}\sigma &= \sigma_E + \frac{1}{2A}\sum_i T_{Bio}G_{\sigma i}H_{\sigma i}\\ \tau &= \tau_B + \tau_E + \frac{1}{2A}\sum_i T_{Bio}G_{\tau i}H_{\tau i}\end{aligned}\right| \quad . \quad . \quad . \quad . \quad . \quad . \quad (314)$$

where,

$$\left.\begin{aligned}G_{\sigma i} &= \frac{g_i\mu_i}{\rho t_s^*}\Big/\frac{T_{Bio}}{2A}\oint_S \frac{h_i}{t}\,ds\\ G_{\tau i} &= -\frac{1}{t^*}\frac{dg_i}{dr}\Big/\frac{T_{Bio}}{2A}\oint_S \frac{h_i}{t}\,ds\end{aligned}\right| \quad . \quad . \quad . \quad . \quad . \quad (315)$$

and the H-functions are exactly as given in Part 4 for the case of pure torque.

$$\left.\begin{aligned} H_{\sigma i} &= \mu_i^{-1}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)\oint_S \frac{h_i}{t}\,ds \\ H_{ri} &= \frac{h_i}{t}\oint_S \frac{h_i}{t}\,ds \end{aligned}\right| \quad . \quad . \quad . \quad . \quad . \quad . \quad (138)$$

T_{Bio} is the value of T_{Bi} at a fixed section preferably the root.

In certain special cases some of the H-functions vanish while the corresponding centres of twist S_i are at infinity. Then, in the presence of shear loads, T_{Bio} becomes infinite and the formulæ (314) require modification as will be shown in 5.4.12.

Of particular interest is the shear stress distribution at the built-in end. From equations (311) and (315),

$$[G_i]_{\rho=1} = -1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (315a)$$

and hence from equation (314),

$$[\tau]_{\rho=1} = \tau_B + \tau_E - \frac{1}{2A}\sum_i T_{Bio}H_{ri} \quad . \quad . \quad . \quad . \quad . \quad (314a)$$

This equation may be compared with (139), and once more verifies the more general theory given in 2.3.

5.4.5. *The conical tube with $t^* = \rho^\alpha$ and $t_i^* = \rho^\beta$; the particular case when $\alpha = \beta$.*

In this case equation (301) reduces to,

$$r^2\frac{d^2g_i}{dr^2} + (1-\alpha)\,r\,\frac{dg_i}{dr} - \mu_i^2 r_o^2 \rho^{\alpha-\beta} g_i = -\frac{r_o^2}{2A}\oint_S \frac{h_i}{t}\,ds\,.\left\{\frac{T_{Bi}}{r}(1+\alpha) - \frac{dT_{Bi}}{dr}\right\} \quad . \quad (316)$$

which should be compared with equation (155). The complementary functions Ψ_1 and Ψ_2 of this equation are given by equation (84). Denoting by V_i the R.H.S. of equation (316) the complete solution of this equation may be written,

$$g_i = \Psi_1\int \frac{V_i\Psi_2}{r^2(\Psi_1'\Psi_2 - \Psi_2'\Psi_1)}\,dr - \Psi_2\int \frac{V_i\Psi_1}{r^2(\Psi_1'\Psi_2 - \Psi_2'\Psi_1)}\,dr \quad . \quad . \quad (156a)$$

which is analogous to equation (156) for the pure torque case.

In the particular case where $\alpha = \beta$ equation (316) reduces to,

$$r^2\frac{d^2g_i}{dr^2} + (1-\alpha)\,r\,\frac{dg_i}{dr} - \mu_i^2 r_o^2 g_i = -\frac{r_o^2}{2A}\oint_S \frac{h_i}{t}\,ds\,.\left\{\frac{T_{Bi}}{r}(1+\alpha) - \frac{dT_{Bi}}{dr}\right\} \quad . \quad (317)$$

which should be compared with equation (157). The complementary function of equation (317) is given by equation (86), namely,

$$g_i = r^{\alpha/2}\{D_1 r^{\nu_i} + D_2 r^{-\nu_i}\} \quad . \quad . \quad . \quad . \quad . \quad (86)$$

where

$$\nu_i = \frac{1}{2}[\alpha^2 + 4\mu_i^2 r_o^2]^{1/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (86a)$$

5.4.6. *The conical tube with $t^* = t_s^* = \rho^\alpha$ under a concentrated load at $r = r_1$.*

Any concentrated load on a conical tube may be represented by the components S_x and S_y acting through the point $E'(r_1, \rho_1\xi_E, \rho_1\eta_E)$, in the directions XO and YO.

Now for $r_i \leqslant r \leqslant r_o$,

$$T = -S_x \rho_1 \eta_E + S_y \rho_1 \xi_E$$

$$Q_x = S_x \frac{r_1}{r} \text{ and } Q_y = S_y \frac{r_1}{r}$$

Hence from equation (300b),

for $r_c \leqslant r \leqslant r_i$

$$T_{Bi} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (318a)$$

and for $r_i \leqslant r \leqslant r_o$

$$T_{Bi} = T_{Bio} = S_x \rho_1 (\eta_i - \eta_E) - S_y \rho_1 (\xi_i - \xi_E) \quad . \quad . \quad . \quad . \quad (318b)$$

It follows from equations (318) that for each mode i the applied load is equivalent to a constant torque T_{Bio} acting at the section $r = r_1$. Now the stress distribution for a concentrated torque has been investigated in 4.1.4 and the results obtained there may be immediately applied here.

Thus rewriting equations (314) for the present case one obtains,

for $r_c \leqslant r \leqslant r_i$,

$$\left.\begin{aligned} \sigma' &= \frac{1}{2A} \sum_i T_{Bio} G'_{\sigma i} H_{\sigma i} \\ \tau' &= \frac{1}{2A} \sum_i T_{Bio} G'_{\tau i} H_{\tau i} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (319a)$$

and for $r_i \leqslant r \leqslant r_o$,

$$\left.\begin{aligned} \sigma &= \sigma_E + \frac{1}{2A} \sum_i T_{Bio} G_{\sigma i} H_{\sigma i} \\ \tau &= \tau_E + \frac{1}{2A} \sum_i T_{Bio} G_{\tau i} H_{\tau i} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (319b)$$

where T_{Bio} is given by equation (318b) and the G-functions by equations (137a) and (137b) of 4.1.4. Note that the h_x and h_y-functions in τ_E have to be calculated for zero torque about the point E by conditions (282).

This analysis shows that the G-functions for $\alpha = 0$ given in Figs. 7-10 (pp. 240-243) may be immediately applied in the present case by replacing in the titles the word torque by shear.

The stresses for any distributed shear may be obtained by integrating from equations (319).

5.4.7. *The built-in conical tube with* $t^* = t_x^* = \rho^n$ *under distributed load* $\omega_x = \omega_{xo} \rho^{\gamma - 2}$.

The case of a conical tube under a distributed load,

$$\omega_x = \omega_{xo} \rho^{\gamma - 2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (320)$$

acting through a line OE will now be investigated. The corresponding results for a load ω_y may be obtained immediately from the present analysis. The form (320) includes the practically important cases of a uniformly distributed load ($\gamma = 2$), and of a distributed load with intensity proportional to the chord ($\gamma = 3$).

From equation (320),

$$\left.\begin{aligned} S_x &= \frac{\omega_{x0} r_0}{\gamma - 1}\,(\rho^{\gamma-1} - \rho_c{}^{\gamma-1}) \\ M_y &= \frac{\omega_{x0} r_0{}^2}{\gamma\,(\gamma-1)}\,\{\,(\rho^\gamma - \rho_c{}^\gamma) - \gamma\,(\rho - \rho_c)\,\rho_c{}^{\gamma-1}\,\} \\ Q_x &= S_x - \frac{M_y}{r} = \frac{\omega_{x0} r_0}{\gamma}\,\frac{\rho^\gamma - \rho_c{}^\gamma}{\rho} \end{aligned}\right\} \quad . \quad . \quad . \quad (321)$$

and the torque about the Z-axis,

$$T = -\,Q_x \rho \eta_E = -\,\frac{\omega_{x0} r_0 \eta_E}{\gamma}\,(\rho^\gamma - \rho_c{}^\gamma) \quad . \quad . \quad . \quad . \quad . \quad (322)$$

Hence from equation (300b),

$$T_{Bi} = Q_x \rho\,(\eta_i - \eta_E) = \frac{\omega_{x0} r_0\,(\eta_i - \eta_E)}{\gamma}\,(\rho^\gamma - \rho_c{}^\gamma) \quad . \quad . \quad . \quad . \quad (323)$$

or

$$T_{Bi} = T_{Bio}\,\frac{\rho^\gamma - \rho_c{}^\gamma}{1 - \rho_c{}^\gamma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (323a)$$

where

$$T_{Bio} = \frac{\omega_{x0} r_0\,(\eta_i - \eta_E)}{\gamma}\,(1 - \rho_c{}^\gamma) \quad . \quad . \quad . \quad . \quad . \quad . \quad (323b)$$

For each mode i the torque T_{Bi} can be represented as the sum of the variable torque,

$$\frac{T_{Bio}}{1 - \rho_c{}^\gamma}\,\rho^\gamma$$

and the constant torque

$$-\,\frac{T_{Bio}}{1 - \rho_c{}^\gamma}\,\rho_c{}^\gamma$$

The analysis for a torque proportional to ρ^γ has been given in section 4.2.4 and for a constant torque in 4.1.4 with $\rho_1 = \rho_c$. Thus the G-functions for the present case may be obtained from equations (166) and (137) by means of the equation,

$$G\ (\text{present case}) = \frac{1}{1 - \rho_c{}^\gamma}\,\{\,G\,(166) - \rho_c{}^\gamma G\,(137)\,\} \quad . \quad . \quad . \quad (324)$$

Using equations (314) and (323a) the equations for the stresses may be written,

$$\left.\begin{aligned} \sigma &= \sigma_E + \frac{\omega_{x0} r_0\,(1 - \rho_c{}^\gamma)}{2A\gamma}\,\sum_i\,(\eta_i - \eta_E)\,G_{\sigma i} H_{\sigma i} \\ \tau &= \tau_E + \frac{\omega_{x0} r_0\,(1 - \rho_c{}^\gamma)}{2A\gamma}\,\sum_i\,(\eta_i - \eta_E)\,G_{\tau i} H_{\tau i} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (325)$$

where the H-functions are given in equations (138) and the G-functions in equations (326) and (327) of the following page.

Note that the h_x and h_y-functions in τ_E have to be calculated for zero torque about the point E by conditions (282).

It may be noted that when $\gamma = 1$ equation (323b) becomes

$$T_{Bi} = T_{Bio}\,\frac{\rho - \rho_c}{1 - \rho_c}$$

which is of the same form as equation (168). Hence Figs. 11 and 12 of pp. 250 and 251 are also applicable to the case when

$$\omega_x = \omega_{x0} \rho^{-1}$$

$$G_{s1} = +\rho^{-(\alpha+2)/2}\left\{\bar{D}_{11}\rho^{\nu_1} + \bar{D}_{21}\rho^{-\nu_1} - \frac{1}{1-\rho_e{}^{\gamma}}\left[\frac{\gamma-(1+z)}{\mu_1{}^2r_0{}^2-[\gamma-(1+z)](\gamma-1)}\rho^{\gamma} + \frac{1+z}{\mu_1{}^2r_0{}^2-(1+z)}\rho_e{}^{\gamma}\right]\rho^{-(\alpha+2)/2}\right\}\mu_1 r_0$$

$$G_{r1} = -\rho^{-(\alpha+2)/2}\left\{\bar{D}_{11}\left(\frac{\alpha}{2}+\nu_1\right) + \bar{D}_{21}\left(\frac{\alpha}{2}-\nu_1\right)\rho^{-\nu_1} - \frac{1}{1-\rho_e{}^{\gamma}}\left[\frac{[\gamma-(1+z)](\gamma-1)}{\mu_1{}^2r_0{}^2-[\gamma-(1+z)](\gamma-1)}\rho^{\gamma} - \frac{1+z}{\mu_1{}^2r_0{}^2-(1+z)}\rho_e{}^{\gamma}\right]\rho^{-(\alpha+2)/2}\right\}$$

(326)

where,

$$\bar{D}_{11} = -\left(\frac{\alpha}{2}-\nu_1\right)\rho_e{}^{\gamma}\frac{\gamma\{\mu_1{}^2r_0{}^2-(1+z)[\gamma-(1+z)]\}\rho_e{}^{-(\alpha+2)/2}+\left(\frac{\alpha}{2}+\nu_1\right)[\{\mu_1{}^2r_0{}^2-(1+z)\}\rho_e{}^{-\gamma}-\{\mu_1{}^2r_0{}^2-[\gamma-(1+z)](\gamma-1)\}]\rho_e{}^{-\nu_1}}{\{\mu_1{}^2r_0{}^2-[\gamma-(1+z)](\gamma-1)\}\{\mu_1{}^2r_0{}^2-(1+z)\}(1-\rho_e{}^{\gamma})\left\{\left(\frac{\alpha}{2}+\nu_1\right)\rho_e{}^{-\nu_1}-\left(\frac{\alpha}{2}-\nu_1\right)\rho_e{}^{\nu_1}\right\}}$$

$$\bar{D}_{21} = +\left(\frac{\alpha}{2}+\nu_1\right)\rho_e{}^{\gamma}\frac{\gamma\{\mu_1{}^2r_0{}^2-(1+z)[\gamma-(1+z)]\}\rho_e{}^{-(\alpha+2)/2}+\left(\frac{\alpha}{2}-\nu_1\right)[\{\mu_1{}^2r_0{}^2-(1+z)\}\rho_e{}^{\gamma}-\{\mu_1{}^2r_0{}^2-[\gamma-(1+z)](\gamma-1)\}]\rho_e{}^{\nu_1}}{\{\mu_1{}^2r_0{}^2-[\gamma-(1+z)](\gamma-1)\}\{\mu_1{}^2r_0{}^2-(1+z)\}(1-\rho_e{}^{\gamma})\left\{\left(\frac{\alpha}{2}+\nu_1\right)\rho_e{}^{-\nu_1}-\left(\frac{\alpha}{2}-\nu_1\right)\rho_e{}^{\nu_1}\right\}}$$

(327)

5.4.8. *The cylindrical tube with linearly varying thicknesses.*

The g_i functions are as in equation (173) of 4.2.6. except that the function V is now given by,

$$V_i = -\frac{1}{2A}\oint_S \frac{h_i}{t}\,ds\,.\,\left\{\frac{T_{Bi}}{z'+d} - \frac{dT_{Bi}}{dz'}\right\} \quad . \quad . \quad . \quad . \quad . \quad (328)$$

The two arbitrary constants in equation (173) can be obtained from the boundary conditions (310a)—(313a) where t_s^* and t^* are given by equations (89) with $\alpha=\beta=1$.

5.4.9. *The cylindrical tube of constant thickness longitudinally under a concentrated load at $z'=z_1$.*

Any concentrated load on a cylindrical tube may be represented by the components S_x and S_y acting in the directions XO, YO through the point E' in the diaphragm at $z'=z_1$.

From equation (302a), or (318) with $\rho_i=1$,

for $0\leqslant z'\leqslant z_1$,

$$T_{Bi}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (329a)$$

and for $z_1\leqslant z'\leqslant c$,

$$T_{Bi}=T_{Bio}=S_x(\eta_i-\eta_E)-S_y(\xi_i-\xi_E) \quad . \quad . \quad . \quad . \quad (329b)$$

It follows from equations (329) that for each mode i the applied load is equivalent to a constant torque T_{Bio} acting at the section $z'=z_1$. Now the stress distribution for a concentrated torque has been investigated in 4.1.5, and the results obtained there may be immediately applied here. Thus rewriting equations (314) for the present case one obtains,

for $0\leqslant z'\leqslant z_1$,

$$\left.\begin{aligned}\sigma' &= -\frac{1}{2A}\sum_i T_{Bio}G'_{\sigma i}H_{\sigma i}\\ \tau' &= \frac{1}{2A}\sum_i T_{Bio}G'_{\tau i}H_{\tau i}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (330a)$$

and for $z_1\leqslant z'\leqslant c$,

$$\left.\begin{aligned}\sigma &= \sigma_E+\frac{1}{2A}\sum_i T_{Bio}G_{\sigma i}H_{\sigma i}\\ \tau &= \tau_E+\frac{1}{2A}\sum_i T_{Bio}G_{\tau i}H_{\tau i}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (330b)$$

where T_{Bio} is given by equation (329b) and the G-functions by equations (142a) and (142b) of 4.1.5. Note that the h_x and h_y functions in τ_E have to be calculated for zero torque about the point E by conditions (282). The plots of the G_σ and G_τ functions for the cylindrical tube given in Figs. 7 to 10 are again applicable in the present case.

The stresses for any distributed shear may be obtained by integrating from equations (330).

5.4.10. *The cylindrical tube of constant thickness under uniformly distributed load ω_x between the root and $z'=z_1$.*

The case of a cylindrical tube of constant thickness under a uniformly distributed load ω_x acting from $z'=z_1$ to $z'=c$ through a line OE parallel to the generators will now be investigated. The corresponding results for a distribution ω_y may be obtained immediately from the present analysis.

From equation (302a) , or from (329) by integration,

for $0\leqslant z'\leqslant z_1$,

$$T_{Bi}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (331a)$$

and for $z_1 \leqslant z' \leqslant c$,

$$T_{Bi} = \omega_x (z' - z_1)(\eta_i - \eta_E) \quad . \quad . \quad . \quad . \quad . \quad . \quad (331b)$$

Equation (331b) may be rewritten,

$$T_{Bi} = T_{Bio} \frac{z' - z_1}{c - z_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (331c)$$

where

$$T_{Bio} = \omega_x (c - z_1)(\eta_i - \eta_E) \quad . \quad . \quad . \quad . \quad . \quad (331d)$$

It follows from equations (331) that for each mode i the applied load is equivalent to a torque T_{Bi} varying linearly from zero at $z' = z_1$ to T_{Bio} at $z' = c$. Now the stress distribution for such a torque variation has been investigated in 4.2.7 and the results obtained there may be immediately applied here.

The stresses are as in equations (330), where T_{Bio} is given by equation (331d) and the G-functions by equations (180a) and (180b) of 4.2.7. Note that the h_x and h_y functions in τ_E have to be calculated for zero torque about the point E by conditions (282). When the load is distributed over the entire tube the G-functions for the cylindrical tube shown in Figs. 11 to 12 are again applicable to the present case.

5.4.11. *The cylindrical tube of constant thickness longitudinally under arbitrary shear loading.*

The shear loading is assumed to arise from a load system acting through a line OE parallel to the generators. Only S_x need be considered since the results for S_y follow immediately.

An arbitrary shear distribution may be represented by the Fourier series,

$$S_x = \sum_{k \text{ odd}}^{\infty} S_k \cos \frac{k\pi z'}{2c} \quad . \quad . \quad . \quad . \quad . \quad . \quad (332)$$

By taking a sufficient number of terms this series will represent any shear distribution. The theoretical zero at $z' = c$ necessitates, however, a large number of terms to represent adequately the shear near the root. To avoid this it is best to work with the Fourier series for $S_x - S_{xo}$ where S_{xo} is the shear at the root, and to combine the distribution $S_x - S_{xo}$ with the tip shear case included in the analysis of 5.4.9. Any other discontinuities, due to concentrated loads, may be removed in a similar manner.

It is only necessary to solve for the typical term,

$$S_k \cos \frac{k\pi z'}{2c}$$

of the distribution $S_x - S_{xo}$.

From equation (302a), or from (329) by integration,

$$T_{Bi} = S_k (\eta_i - \eta_E) \cos \frac{k\pi z'}{2c} \quad . \quad . \quad . \quad . \quad . \quad (333)$$

Equation (333) may be rewritten,

$$T_{Bi} = T_{ki} \cos \frac{k\pi z'}{2c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (333a)$$

where,

$$T_{ki} = S_k (\eta_i - \eta_E) \quad . \quad . \quad . \quad . \quad . \quad . \quad (333b)$$

It follows from equations (333) that for each mode i the applied load is equivalent to a torque

$$T_{ki} \cos \frac{k\pi z'}{2c}$$

Now the stress distribution for such a torque variation has been investigated in 4.2.8 and the results there obtained may be immediately applied here.

Thus from equations (184) and (333b) the expressions for the stresses due to the shear distribution $S_x - S_{x0}$ are,

$$\left.\begin{aligned}\sigma &= \sigma_E - \frac{1}{2A}\sum_{k\ \text{odd}}^{\infty} S_k \left(\frac{2c}{k\pi}\right) \sin\frac{k\pi z'}{2c} \sum_i \frac{k^2\pi^2}{4\mu_i^2 c^2 + k^2\pi^2}(\eta_i - \eta_E)\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)\oint_S \frac{h_i}{t}\,ds \\ \tau &= \tau_E + \frac{1}{2A}\sum_{k\ \text{odd}}^{\infty} S_k \cos\frac{k\pi z'}{2c} \sum_i \frac{k^2\pi^2}{4\mu_i^2 c^2 + k^2\pi^2}(\eta_i - \eta_E)\frac{h_i}{t}\oint_S \frac{h_i}{t}\,ds\end{aligned}\right\} \quad . \quad (334)$$

The same remark applies to the calculation of τ_E as in preceding paragraphs. As stated in 4.2.8 the results obtained by Hemp, 1943[12], for a doubly symmetrical rectangular tube with booms appear as a particular case of equations (334).

5.4.12. *The particular case of eigenloads giving bending without twist.*

As stated at the end of 5.4.4 the case when in a mode i,

$$\oint_S \frac{h_i}{t}\,ds = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (335)$$

while the corresponding centre of twist S_i is at infinity requires special consideration. For example, the expressions for the stresses in a tube loaded in a plane of symmetry involve only eigenloads of this particular type, which give only flexural deformation of the tube. On the other hand, if the tube is loaded normally to the plane of symmetry and/or by pure torque, eigenloads not in general satisfying condition (335) are involved, with centres of twist S_i lying in the plane of symmetry. However, if there is a further plane of symmetry the latter group of eigenloads will break up into two distinct sets; the first satisfies condition (335) with centres of twist at infinity in the first plane of symmetry; the second corresponds to twisting modes about the centre of symmetry. Eigenloads for which condition (335) holds while S_i is at finitum are automatically excluded since the corresponding terms in the stress expressions vanish.

It follows from the above considerations that, when there are one or two planes of symmetry, the secular equation will break up into two or three factors separating the different groups of eigenloads (see also 5.7.5).

To fix ideas, consider the case of a tube with a plane of symmetry through $G\bar{Y}$. For the eigenloads corresponding to pure flexural deformation in the plane of symmetry,

$$\oint_S \frac{h_i}{t}\,ds = 0 \text{ and } \xi_i = \infty,$$

but the product

$$\xi_i \oint_S \frac{h_i}{t}\,ds$$

has a finite value. This product appears as one of the unknowns in the set of homogeneous linear simultaneous equations leading to the secular equation (see pp. 220-223 of 3.2.1). In practical calculations these modes can be separated *ab initio* from the remaining ones by excluding the unknowns

$$\oint_S \frac{h_i}{t}\,ds \text{ and } \eta_i \oint_S \frac{h_i}{t}\,ds$$

and omitting two of the conditions (54) *i.e.*,

$$\oint h_i p\,ds = \oint h_i \sin\psi\,ds = 0$$

which are automatically satisfied.

To obtain the g_i-functions corresponding to the loads in the plane of symmetry it is only necessary to substitute in the differential equations (299) or (301) and the boundary conditions (310) to (313)

$$-Q_y \rho a_{xi} \text{ in place of } T_{Bi} \oint_S \frac{h_i}{t}\, ds$$

where,

$$a_{xi} = \xi_i \oint_S \frac{h_i}{t}\, ds \qquad (336)$$

In fact the differential equations (299) become,

$$\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_i}{dr}\right) - \mu_i^2 \frac{g_i}{\rho t_s^*} = -\frac{a_{xi}}{2A}\frac{d}{dr}\left(\frac{Q_y}{t^*}\right) \qquad (337)$$

for the conical tube,

and

$$\frac{d}{dz'}\left(\frac{1}{t^*}\frac{dg_i}{dz'}\right) - \mu_i^2 \frac{g_i}{t_s^*} = -\frac{a_{xi}}{2A}\frac{d}{dz'}\left(\frac{S_y}{t^*}\right) \qquad (337a)$$

for the cylindrical tube.

A relation between v_i and a_{xi} corresponding to the relation, given in equation (35), between θ_i and $\oint_S \frac{h_i}{t}\, ds$ will now be shown. From equation (30),

$$\frac{d}{dr}\left(\frac{v_i}{\rho}\right) = -\xi_i \frac{d\theta_i}{dr}$$

Using (35) and (336) in the latter equation one obtains for the conical tube,

$$\frac{d}{dr}\left(\frac{v_i}{\rho}\right) = -\frac{a_{xi}}{2AG}\frac{1}{\rho t^*}\frac{dg_i}{dr} \qquad (338)$$

and for the cylindrical tube,

$$\frac{dv_i}{dz'} = -\frac{a_{xi}}{2AG}\frac{1}{t^*}\frac{dg_i}{dz'} \qquad (338a)$$

The general expressions for the stresses given in equation (314) reduce, for a loading acting in the plane of symmetry through $G\bar{Y}$, to,

$$\left.\begin{aligned} \sigma &= \sigma_Y - \frac{Q_{yo}}{2A}\sum_i a_{xi} G_{\sigma i} L_{\sigma i} \\ \tau &= \tau_Y - \frac{Q_{yo}}{2A}\sum_i a_{xi} G_{\tau i} L_{\tau i} \end{aligned}\right\} \qquad (339)$$

where the summation extends over the modes i satisfying condition (335), and the L-functions are,

$$\left.\begin{aligned} L_{\sigma i} &= \mu_i^{-1}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right) \\ L_{\tau i} &= \frac{h_i}{t} \end{aligned}\right\} \qquad (340)$$

If the entire loading acts in the plane of symmetry no further eigenloads need be considered, since in these modes $T_{Bi}=0$. Any general loading may be reduced to a loading in the plane of symmetry plus shears normal to this plane and/or a torque distribution. The latter loadings may be analysed as in the general case and will involve eigenfunctions different from those occurring in equations (337) to (340).

The procedure when there are two planes of symmetry will be obvious to the reader.

To illustrate further the above analysis consider the single-cell sections for which

$$p_E t = \text{const.}.$$

As stated in 2.2 such sections do not warp under torque and the Bredt-Batho formula holds for any distribution of torque. It follows that condition (335) is satisfied for all eigenloads. However, for some or all eigenloads S_i will be at infinity and the corresponding modes will give rise to axial constraint stresses due to bending. For example, the stresses due to loads in the $G\bar{Y}$ plane are given by equation (339). Since under torque such tubes twist about OE the point E is by definition the shear centre E_S, which in this case is independent of the t_s distribution. These arguments apply also to multi-cell tubes satisfying the above condition over each wall and for which at each wall-joint,

$$\Sigma p_E t = 0.$$

In this relation the sign of p_E is determined by taking the origin of s for each wall at the common joint.

TO BE CONTINUED.

AN APPRECIATION

The Council of the Royal Aeronautical Society wish to thank those Companies who, by their generous co-operation, have done so much to help in the production of the Journal

ACCLES & POLLOCK LIMITED

A&P
Make it with Tubes

AIRWORK LIMITED

AIRWORK
LIMITED

AEROPLANE & MOTOR ALUMINIUM CASTINGS LTD.

ALUMINIUM GRAVITY DIE CAST CYLINDER HEAD

ARMSTRONG SIDDELEY MOTORS LTD.

ARMSTRONG SIDDELEY

AIRCRAFT MATERIALS LIMITED

AML
STRUCTURAL MATERIALS and COMPONENTS

SIR W. G. ARMSTRONG WHITWORTH AIRCRAFT LTD.

SIR W. G.
ARMSTRONG WHITWORTH
AIRCRAFT LIMITED

AIRSPEED LIMITED

AIRSPEED

AUSTER AIRCRAFT LIMITED

THE STEEL
AUSTER
AEROPLANE
"THE EYES OF THE ARMY"

AUTOMOTIVE PRODUCTS COMPANY LTD.

HIGH PRECISION HYDRAULICS
Lockheed

BLACKBURN AIRCRAFT LTD.

Blackburn
AIRCRAFT

AVIMO LIMITED

AVIMO

OPTICAL - MECHANICAL - ELECTRICAL INSTRUMENTS
AERONAUTICAL EQUIPMENT

JAMES BOOTH & COMPANY LTD.

BOOTH
BIRMINGHAM
DURALUMIN
REGISTERED TRADE MARK

BAKELITE LIMITED

TREFOIL
BAKELITE PLASTICS
REGD. TRADE MARKS

BOULTON PAUL AIRCRAFT LTD.

BPA

T. M. BIRKETT & SONS LTD.

BIRSO
TRADE MARK

NON-FERROUS CASTINGS
AND MACHINED PARTS
HANLEY - - STAFFS

BRAKE LININGS LIMITED

DURON
REGISTERED
BRAKE AND CLUTCH LININGS

BIRMETALS LTD.

BIRMETALS

THE BRISTOL AEROPLANE CO., LTD.

"Bristol"

THE BRITISH ALUMINIUM CO., LTD

BROOM & WADE LTD.

"BROOMWADE"

AIR COMPRESSORS
AND PNEUMATIC TOOLS

THE BRITISH AVIATION INSURANCE CO. LTD

BRITISH AVIATION SERVICES LTD.

BROWN BROTHERS (AIRCRAFT) LTD.

Brown Brothers (AIRCRAFT) Ltd.

CONTROLS · AEROJACKS · A.G.S. & A.S. PARTS

BRITISH INSULATED CALLENDER'S CABLES LTD.

BUTLERS LIMITED

AUTOMOBILE, AIRCRAFT AND MARITIME LAMPS
SEARCHLIGHTS AND MOTOR ACCESSORIES

BRITISH THOMSON-HOUSTON CO., LIMITED (THE)

Magnetos and Electrical Equipment

THE CHLORIDE ELECTRICAL STORAGE CO. LTD.

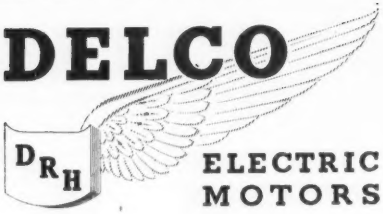
Exide

AIRCRAFT BATTERIES

BRITISH WIRE PRODUCTS LTD.

THE DE HAVILLAND AIRCRAFT COMPANY LTD.

DELCO-REMY & HYATT LTD



THE FAIREY AVIATION COMPANY LTD.

DOWTY EQUIPMENT LIMITED

FIRTH-VICKERS STAINLESS STEELS LTD.

DUNFORD & ELLIOTT (SHEFFIELD) LTD.

FOLLAND AIRCRAFT LIMITED

DUNLOP RUBBER CO., LTD.

GENERAL AIRCRAFT LIMITED

ENGLISH STEEL CORPORATION LTD.

GLOSTER AIRCRAFT CO., LTD.

HANDLEY PAGE LIMITED

H. M. HOBSON LTD.

Hobson

HAWKER AIRCRAFT LIMITED

HAWKER

F. A. HUGHES & CO., LIMITED

Distributors of

ELEKTRON

REGISTERED TRADE MARK

MAGNESIUM ALLOYS

HELLIWELLS LIMITED

HENRY HUGHES & SON LTD

THE HESTON AIRCRAFT CO., LTD.

THE HUGHES-JOHNSON STAMPINGS LTD.

HUNTING AVIATION LIMITED

KELVIN BOTTOMLEY & BAIRD LTD.

IMPERIAL CHEMICAL INDUSTRIES LTD.

LIGHT-METAL FORGINGS LTD

INTEGRAL LIMITED

INTEGRAL
HYDRAULIC PUMPS
AND EQUIPMENT

LODGE PLUGS LIMITED

LODGE
PLUGS

IRVING AIR CHUTE OF GREAT BRITAIN LTD

MARSTON EXCELSIOR LIMITED

K.L.G. SPARKING PLUGS LTD.

MILES AIRCRAFT LIMITED

Miles
AIRCRAFT

NORMALAIR LIMITED

NORMALAIR

MOLLART ENGINEERING COMPANY LTD.

ME

Members of the Gauge and
Tool Makers' Association

THE PALMER TYRE LIMITED

'First in the Air'

Palmer

THE PALMER TYRE LIMITED
Herga House, Vincent Square, London, S.W.1.

THE MOND NICKEL COMPANY LTD

NICKEL

PERCIVAL AIRCRAFT LIMITED

D NAPIER & SON LIMITED

N

PETO & RADFORD

DAGENITE
AIRCRAFT BATTERIES

PORTSMOUTH AVIATION LIMITED

R.F.D. COMPANY LTD.

PRESSED STEEL CO., LIMITED

PRESTCOLD
Refrigeration

A. V. ROE & COMPANY LIMITED

THE PULSOMETER ENGINEERING CO. LTD.

ROLLS-ROYCE LIMITED

ROLLS-ROYCE
AERO-ENGINES

QANTAS EMPIRE AIRWAYS

ROTAX LIMITED

ROTAX
AIRCRAFT ELECTRICAL ENGINEERS

ROTOL LIMITED

SHORT BROTHERS (ROCHESTER & BEDFORD) LIMITED

Shorts

Established 1908

THE FIRST MANUFACTURERS OF AIRCRAFT IN THE WORLD

SANGAMO WESTON LTD.

SKYHI LIMITED

SAUNDERS-ROE LIMITED

SMITHS AIRCRAFT INSTRUMENTS LTD.

SERCK RADIATORS LIMITED

THE SPERRY GYROSCOPE COMPANY LTD

THE TECHNOLOGICAL INSTITUTE OF GREAT BRITAIN

GB TI

COURSES FOR THE EXAMINATIONS OF THE ROYAL AERONAUTICAL SOCIETY

A. C. WICKMAN LIMITED

A.C. Wickman LIMITED

• COVENTRY • ENGLAND •

SPECIALISTS IN MACHINE TOOLS & SMALL TOOLS FOR THE AIRCRAFT INDUSTRY

THE UNITED STEEL COMPANIES LIMITED

"FLYING FOX" ELECTRIC STEELS

S. FOX & CO. LTD. SHEFFIELD

HENRY WIGGIN & COMPANY LTD

For High Quality

NICKEL ALLOYS

VOKES LIMITED

AMBER CROSS

TRADE MARK

SYMBOL OF THE VOKES SERVICE

WILLIAMSON MANUFACTURING CO., LTD

WESTLAND AIRCRAFT LIMITED

WESTLAND AIRCRAFT LIMITED

Yeovil, Somerset

YORKSHIRE ENGINEERING SUPPLIES LTD.

BRONZE FOUNDRIES

YESEATONIA

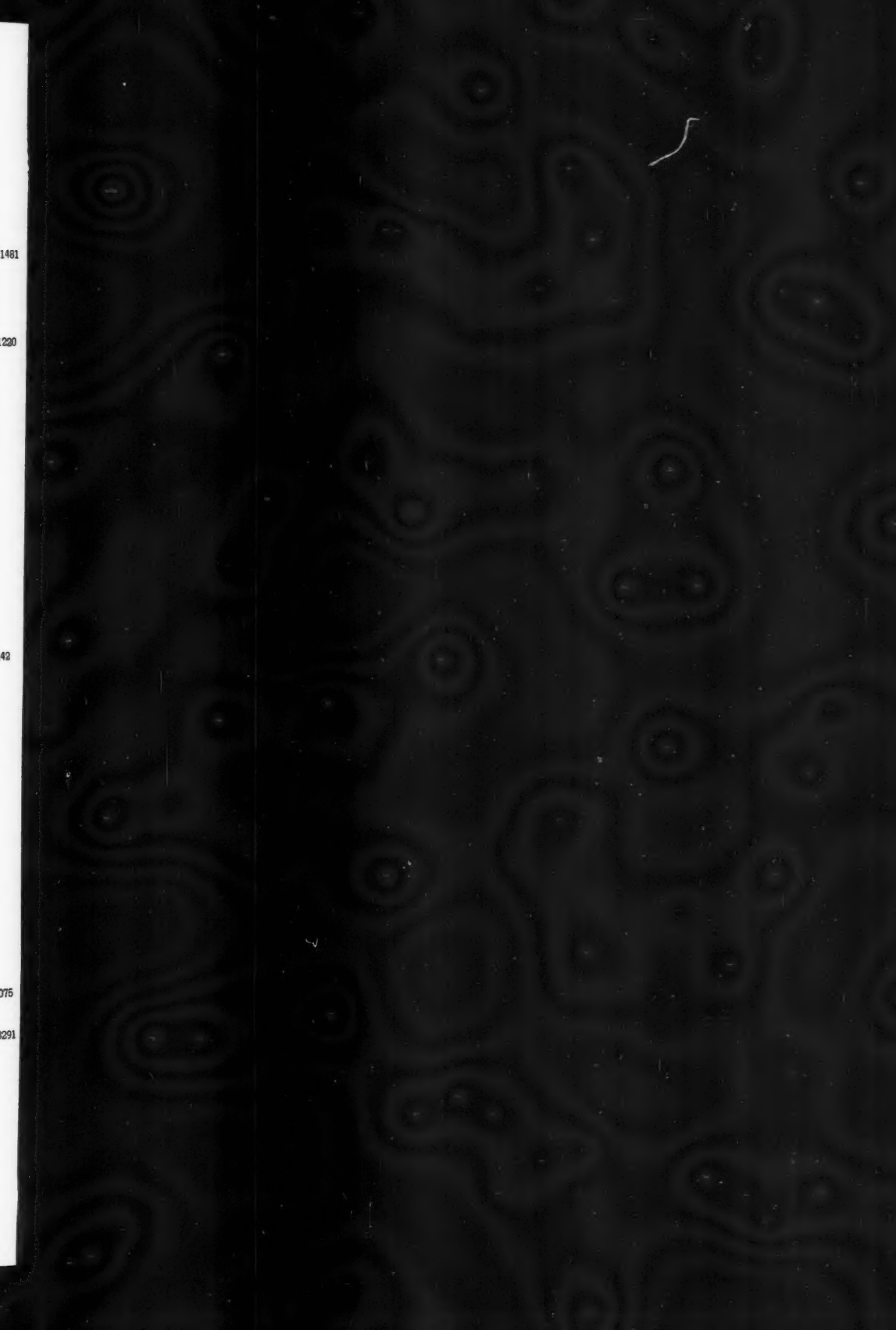
BEARINGS AND BUSHES

DIRECTORY OF ADVERTISERS

ACCLES & POLLOCK LTD.	Paddock Works, Oldbury, Birmingham	Broadwell 1500
AEROPLANE & MOTOR ALUMINIUM CASTINGS LTD.	Wood Lane, Erdington, Birmingham	Erdington 2207-9
AIRCRAFT MATERIALS LTD.	Midland Road, London, N.W.1.	Euston 6151
AIRSPEED LTD.	The Airport, Portsmouth	Portsmouth 74631
AIRWORK LTD.	15 Chesterfield Street, W.1.	Grosvenor 4841
ALBRIGHT & WILSON	49 Park Lane, W.1. Works: Oldbury and Widnes.	Grosvenor 1311
ARMSTRONG SIDDELEY MOTORS LTD. ...	Parkside, Coventry	Coventry 4061
ARMSTRONG WHITWORTH AIRCRAFT, SIR W. G., LTD.	Whitley, Coventry	Coventry 61061-4
AUSTER AIRCRAFT LTD.	Britannia Works, Thurmaston, Leicester	Syston 86106-8
AUTOMATIC COIL WINDER & ELECTRICAL EQUIPMENT CO. LTD.	Winder House, Douglas Street, S.W.1	Victoria 3404-8
AUTOMOTIVE PRODUCTS CO. LTD.	Tachbrook Road, Leamington Spa	Leamington Spa 1700
AVIMO LTD.	Taunton, Somerset	Taunton 3634
BAKELITE LTD.	18 Grosvenor Gardens, S.W.1	Sloane 9911
BIRKETT, T. M. & SONS LTD.	Thomas Street, Hanley, Stoke-on-Trent	Stoke-on-Trent 2184
BIRMETALS LTD	Woodgate Works, Quinton, Birmingham, 32 ...	Woodgate 2253
BLACKBURN AIRCRAFT LTD.	Brough, E. Yorks	Brough 121
BOOTH, JAMES, & CO. LTD.	Argyle Street Works, Nechells, Birmingham, 7 ...	Birmingham E. 1221
BOULTON PAUL AIRCRAFT LTD.	Wolverhampton, Staffs	Fordhouses 3191
BRAKE LININGS LTD.	Buxton, Derbyshire	Buxton 776
BRISTOL AEROPLANE CO. LTD., THE ...	Filton House, Bristol	
BRITISH ALUMINIUM CO. LTD.	Salisbury House, London Wall, E.C.2	Clerkenwell 3494
BRITISH AVIATION INSURANCE CO. LTD. ...	3-4 Lime Street, E.C.3	Mansion House 0444
BRITISH AVIATION SERVICES LTD.	1 Great Cumberland Place, W.1	Paddington 7040
BRITISH BELTING AND ASBESTOS LTD. ...	Cleckheaton, Yorks.	Cleckheaton 800
	59 Southwark Street, S.E.1	Hop 0192
BRITISH EUROPEAN AIRWAYS CORPORATION	Headquarters: Northolt Airport, Middlesex	Ruislip 6061
BRITISH INSULATED CALLENDER'S CABLES LTD.	Main Works: Erith, Helsby, Leigh, Lancs. and Prescot	
BRITISH THOMSON-HOUSTON CO. LTD. ...	Lower Ford Street, Coventry	Coventry 4104
BRITISH WIRE PRODUCTS LTD.	Worcester Road, Stourport-on-Severn	Stourport 240
BROOM & WADE LTD.	High Wycombe, Bucks.	High Wycombe 1630
BROWN BROTHERS (AIRCRAFT) LTD. ...	Great Eastern Street, E.C.2	Bishopsgate 7654
BUTLERS LTD.	Atlantic Works, Grange Road, Small Heath, Birmingham, 10	Victoria 2164-6
CHLORIDE ELECTRICAL STORAGE CO. LTD., THE	Exide Works, Clifton Junction, near Manchester ...	Swinton 2011
COSSOR, A. C. LTD.	Highbury Grove, N.5	Canonbury 1234
DE HAVILLAND AIRCRAFT CO. LTD., THE	Hatfield Aerodrome, Herts.	Hatfield 2345
DECCA NAVIGATOR CO. LTD.	1-3 Brixton Road, S.W.9	Reliance 3311
DELCO-REMY & HYATT LTD.	111 Grosvenor Road, S.W.1	Victoria 6242
DOWTY EQUIPMENT LTD.	Cheltenham, Gloucester	Cheltenham 53471
DUNFORD & ELLIOTT (SHEFFIELD) LTD. ...	Attercliffe Wharf Works, Sheffield, 9	Sheffield 41121-4
DUNLOP RUBBER CO. LTD.	Fort Dunlop, Erdington, Birmingham, 24	Erdington 2121
ENGLISH STEEL CORPORATION LTD.	Vickers Works, Sheffield	Sheffield 41071
FAIREY AVIATION CO. LTD.	Hayes, Middlesex	Hayes 3800
FIRTH, THOS., & JOHN BROWN LTD. ...	Atlas & Norfolk Works, Sheffield, 1	Sheffield 20081, 26491
FIRTH-VICKERS STAINLESS STEELS LTD. ...	Staybrite Works, Sheffield	Sheffield 41193
FOLLAND AIRCRAFT LTD.	Hamble, Southampton, Hants.	Hamble 3191
GENERAL AIRCRAFT LTD.	The London Air Park, Feltham, Middlesex ...	Feltham 3636
GLOSTER AIRCRAFT CO. LTD.	Gloster Works and Aerodrome, Hucclecote, Glos.	Gloucester 6294
HANDLEY PAGE LTD.	Cricklewood, N.W.2	Gladstone 8000
HAWKER AIRCRAFT LTD.	Canbury Park Road, Kingston-on-Thames	Kingston 1044
HAWKER SIDDELEY GROUP	70 Pall Mall, S.W.1.	Abbey 2672
HELLIWELLS LTD.	The Airport, Walsall, Staffordshire	Walsall 4553
HESTON AIRCRAFT CO. LTD., THE	Heston Airport, Middlesex	Southall 2321
HOBSON, H. M. LTD.	Hobson Works, Fordhouses, Wolverhampton ...	Fordhouses 2266
HUGHES, F. A &. CO. LTD.	Metals Dept., Abbey House, Baker Street, N.W.1	Welbeck 2332-6
HUGHES, HENRY, & SON LTD.	Husun Works, New North Rd., Barkingside, Essex	Hainault 2601
HUGHES-JOHNSON STAMPINGS LTD., THE ...	Langley Green, Birmingham	Broadwell 1361
HUNTING AVIATION LTD.	P.O. Box 4, Caernarvon, N. Wales	Caernarvon 580
ILIFFE AND SONS LTD.	Dorset House, Stamford Street, S.E.1	Waterloo 3333
IMPERIAL CHEMICAL INDUSTRIES LTD. ...	London, S.W.1	Victoria 4444
INTEGRAL LTD.	Cousins Street, Dudley Road, Wolverhampton ...	Wolverhampton 24984-6
IRVING AIR CHUTE OF GREAT BRITAIN LTD.	Ickneild Way, Letchworth, Herts.	Letchworth 888

DIRECTORY OF ADVERTISERS

K.L.G. Sparking Plugs Ltd.	Putney Vale, S.W.15	Putney 2671
Kelvin, Bottomley & Baird Ltd.	Kelvin Avenue, Hillington, Glasgow, S.W.2	Halfway 3331
	Kelvin Works, Basingstoke	Basingstoke 690
Light-Metal Forgings Ltd.	Oldbury, Birmingham	Broadwell 1152
Lodge Plugs Ltd.	Rugby	Rugby 2076
Marston Excelsior Ltd.	Wolverhampton	Wolverhampton 21481
Miles Aircraft Limited	49-59 Armley Road, Leeds, 12	Armley 38081-5
Mollart Engineering Co. Ltd.	Reading, Berkshire	Reading 60811
Mond Nickel Co. Ltd., The	Kingston By-Pass, Surbiton, Surrey	Elmbridge 3352-4
	Grosvenor House, Park Lane, W.1	Grosvenor 4131
Napier, D., & Son Ltd.	Acton, W.3	Shepherds Bush 1220
Normalair Ltd.	West Hendford, Yeovil, Somerset	Yeovil 1100
Palmer Tyre Ltd., The	Herga House, Vincent Square, S.W.1	Victoria 8323
Percival Aircraft Ltd.	Luton Airport, Luton, Beds.	Luton 2960
Peto & Radford	Chequers Lane, Dagenham, Essex	Rainham 34
Pitman, Sir Isaac, & Sons Ltd.	Parker Street, Kingsway, W.C.2	Holborn 9791
Portsmouth Aviation Ltd.	The Airport, Portsmouth	Portsmouth 74374
Pressed Steel Co. Ltd.	Cowley, Oxford	Oxford 77701
Pulsometer Engineering Co. Ltd., The	Nine Elms Iron Works, Reading; 39 Victoria Street, S.W.1	Tilehurst 67182-4
Qantas Empire Airways	c/o British Overseas Airways Corporation, Airways Terminal, Buckingham Palace Road, S.W.1	Victoria 3126
R.F.D. Company Ltd.	Stoke Road, Guildford, Surrey	Guildford 3232
Roe, A. V., & Co. Ltd.	Newton Heath, Manchester	Failsworth 2020
Rolls-Royce Ltd.	Derby	Derby 2424
Rotax Ltd.	Willesden Junction, N.W.10	Elgar 7777
Rotol Ltd.	Cheltenham Road, Gloucester	Gloucester 4431
Sangamo Weston Ltd.	Great Cambridge Road, Enfield, Middlesex	Enfield 3434 & 1242
Saunders-Roe Ltd.	49 Parliament Street, Westminster, S.W.1	Whitehall 7271
Serck Radiators Ltd.	Warwick Road, Birmingham, 11	Victoria 0531
Short Brothers (Rochester & Bedford) Ltd.	Rochester, Kent	Chatham 2261
Simmonds Aerocessories Ltd.	2 and 3 Norfolk Street, W.C.2	Temple Bar 2373
Skyhi Ltd.	"Skyhi" Works, Worton Rd., Isleworth, Middlesex	Hounslow 2211
Smiths Aircraft Instruments Ltd.	Cricklewood Works, London, N.W.3	Gladstone 3333
Sperry Gyroscope Co. Ltd., The	Great West Road, Brentford, Middlesex	Ealing 6771
Technological Institute of Great Britain Ltd., The	39 Temple Bar House, Fleet Street, E.C.4	Central 5940
Temple Press Ltd.	Bowling Green Lane, E.C.1	Terminus 3636
United Steel Companies Ltd., The	17 Westbourne Road, Sheffield, 10	Sheffield 60081
Vickers-Armstrongs Ltd.	Vickers House, Broadway, Westminster, S.W.1.	Abbey 7777
	Weybridge Works, Weybridge, Surrey	Byfleet 240-243
Vokes Ltd.	Henley Park, nr. Guildford, Surrey	Guildford 62861
Westland Aircraft Ltd.	Yeovil	Yeovil 1100
Wickman, A. C., Ltd.	Tile Hill, Coventry	Tile Hill 66271
Wiggin, Henry, & Co. Ltd.	Wiggin Street, Birmingham	Edgbaston 2245
Williamson Manufacturing Co. Ltd.	Litchfield Gardens, Willesden Green, N.W.10	Willesden 0073-0075
Yorkshire Engineering Supplies Ltd.	Bronze Foundries, Upper Wortley Road, Leeds, 12	Leeds 38234 & 38291



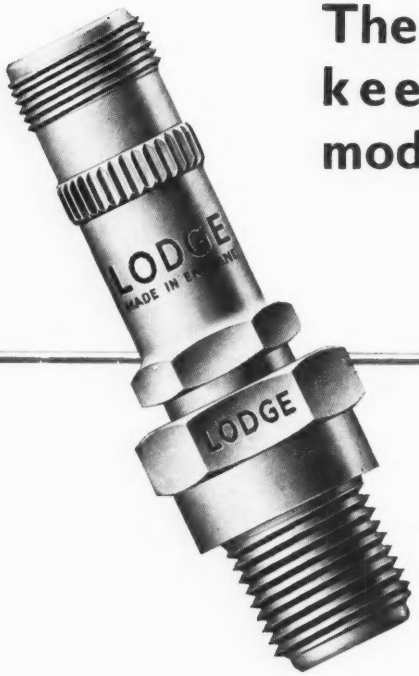
THE GRAVINER COMPANY *retains the services of the inventors and originators of the Graviner Fire-fighting Equipment and by arrangement with* **THE WILKINSON SWORD COMPANY** *the highly specialised manufacturing facilities at the Colnbrook Works have now been extended by the addition of a Development and Scientific Research Department, and additional scientific and engineering development staff have been engaged.*

* * * *

WE *therefore offer to the Aircraft Industry the combined experience of the two companies in the sincere belief that the technical supremacy established in the aircraft fire-fighting equipment field will be maintained, and the equipment will be still further improved to meet present-day requirements so that effective fire protection will be provided for the most modern developments in the Aircraft Industry.*

For maximum power

LODGE



The plug that always keeps abreast of modern engine design

Lodge Plugs Ltd., Rugby.

Printed by the Lewes Press (Wightman & Co. Ltd.), Friars Walk, Lewes, England, and Published by The Royal Aeronautical Society, 4 Hamilton Place, London, W.1, England.

gby.